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The problems associated with uneven forced response vibration amplitudes in bladed disk assemblies is considered in the report. It is established that uneven vibration amplitudes arise principally by the destruction of cyclic-symmetry by some small perturbations usually within the component manufacturing tolerances. Such perturbations first split some of the eigenvalue degeneracies inherent in all cyclic systems. This split in turn gives rise to the modal bifurcation phenomenon. Particular forms of the modal phenomenon give rise to the uneven vibration amplitudes and under some restricted conditions to the mode localization phenomenon. In this report, group theory, singularity theory and singular perturbation theory are combined to give a complete analysis of uneven amplitudes and mode localization; as a prelude to blade vibration control.

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Vibration Dynamics and Control of Bladed Disk Assemblies

Final Report
on Contract AFOSR-[redacted]

Submitted to:

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For budget purposes only, an identical copy of this report is being submitted by Dr. D. Afolabi for contract AFOSR-89-0002 through Indiana University.

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Executive Summary of "Vibration Dynamics and Control of Bladed Disk Assemblies"

This final report documents the work performed at Purdue University during the period of November 1988 to December 1990. The original AFOSR Contracts (#AFOSR-89-0002, AFOSR-89-0014) were written for two years. Consequently this research was partially funded from Professor Nwokahs' PRF grant #670-1667. The objective of the proposed research was to gain a fundamental understanding of how and why periodically configured mechanical and structural systems, (in particular bladed-disk assemblies) with cyclic symmetry and nominally identical sub-structures can display non-uniform amplitudes of vibration when subjected to small but random parameter perturbations that are often within the component manufacturing tolerances. A secondary aim of the proposal was to determine ways of passively/actively (if possible) controlling these uneven vibration amplitudes. This work specifically dealt with the influence of the double (degenerate) eigenvalues present in every cyclic mechanical system and their subsequent splitting under small perturbations, on the uneven vibration amplitudes of the components.

Status:

The work associated with the principal objectives of the project is almost completed and is included in this final report. The procedure for detecting a priori which degenerate eigenvalue pairs will split under given parameter perturbations has been formalized by use of finite group representation theory and is presented in Appendix 1. The procedure for accurately unfolding the singularities induced by the splitting of the double modes of cyclic systems has been formalized by the use of a singular perturbation analysis technique which is valid for any finite order cyclic system and is included in Appendix 2.

The topological basis for the singularities induced by the double modes and the consequences there of are carefully examined and detailed in Appendix 3.

In contra-distinction from recent work in the bladed-disk research literature, numerical studies which show that uneven amplitudes of vibration in perturbed cyclic systems can arise both under strong coupling as well as the weak coupling conditions is included in Appendix 4.

A systematic framework has now been established for a detailed study of perturbed cyclic systems. Future efforts will be aimed at completing any remaining theoretical analysis, development of computational algorithms for such analysis, passive structural redesign to avoid localized high vibration amplitudes and experimental validation of the analytical results.

Publications

Six papers have been developed from this work. The first is essentially Appendix 1. The second and third are included in Appendix 2. The fourth is given in Appendix 3, while the rest are in conference proceedings as given below.

1. Happawana, G.S., Bajaj, A.K., Nwokah, O.D.I., On the dynamics of perturbed symmetric systems. Accepted for presentation and publication in conference proceedings for 13th Biennial ASME Conference on Mechanical Vibration and Noise, September 22-25, 1991, Miami, Florida.

2. Happawana, G.S., Bajaj, A.K., Nwokah, O.D.I., A singular perturbation perspective on mode localization. J. Sound and Vibration, (To appear).

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Personnel

Three faculty members and one graduate student were funded by this contract.

Professor Osita D.I. Nwokah (Resume at the back).

Professor Anil K. Bajaj (Resume at the back).

Professor Dare Afolabi (Resume at the back).

Gemunu S. Happawana (Doctoral Student).

A doctorate degree is expected to be awarded for this work in the next 24 months.

Presentations

Several seminar presentations resulted from this work. Details are contained in the individual Professors' resumes.

1. INTRODUCTION

1.1 Problem Statement

The central aim of the present project has been to:

(i) Gain a fundamental understanding of how and why periodically configured mechanical and structural systems with cyclic symmetry and nominally identical sub-structures can display non-uniform amplitudes of vibration under differential (i.e., small) parameter perturbations that are often within the component manufacturing tolerances.

(ii) Design passive and/or active control mechanisms to overcome such possible uneven amplitudes of vibrations.

1.2 Background and Overview

The study of cyclically configured dynamical systems, otherwise known as bladed-disk assemblies, has been a very active area of research in structural dynamics over the last 25 years. It is a measure of the theoretical difficulties involved in an accurate analysis that at the present time there is no general agreement in the literature as to either the causes of the uneven component vibration amplitudes or as to which component will vibrate with the highest amplitude under parameter variations. This is a sine′ qua-non to establishing benchmark specifications for component vibration control. The work performed under this grant in the last two years clearly indicates that:

(i) Uneven amplitudes of vibration are caused by the modal bifurcation phenomenon or the sensitive dependence of eigenvectors on small parameter variations under some clearly defined conditions. [1,2,3]

(ii) Modal bifurcations in turn are caused by mistuning or small parameter variations from nominal design values, which are often within the component manufacturing tolerances. [4]

(iii) Extreme cases of the uneven amplitudes of component vibrations produce the mode localization phenomenon.

We had shown in the paper in Appendix 3, that very useful qualitative information on the blade mistuning could be obtained by application of the methodology of singularity theory to this problem.

To understand mode localization, one must first study modal bifurcations.

Let $\mu \in \Gamma \subset R^r$, where R^r is an r-dimensional parameter space. If a given structural system has n degrees of freedom, then the characteristic equation for natural frequencies, $\omega^2 = \lambda$, can be written as the n-th order polynomial equation:

$$G(\lambda , \mu) = 0 \text{ , for some } \mu \in \Gamma .$$

It turns out that the characteristic polynomial under appropriate modifications behaves like the potential function in singularity (or catastrophe) theory. [5] Hence the degenerate critical points of $G(\lambda , \mu) = 0$ correspond to the repeated roots (i.e., the repeated eigenvalues) of $G(\lambda , \mu)$. By studying $\frac{\partial G}{\partial \lambda}$ together with appropriate higher order differentials, and $G(\lambda , \mu) = 0$, we can determine the set of all $\mu \in \Gamma$ at which $G(\lambda , \mu)$ has degenerate eigenvalues. This set, which is called the bifurcation (or catastrophe) set, partitions the parameter space Γ into distinct submanifolds whose boundary is the bifurcation set.

We can conclude from the basic theorems and results from singularity theory and catastrophe theory [6,7,8] that the modal behavior of our structural system displays sensitive dependence on parameters only in the neighborhood of the bifurcation set. We have identified two distinct degenerate behavior patterns in structural systems, namely:

a) Coupling induced degeneracy,

b) Geometry or symmetry induced degeneracy.

Furthermore, we have noted that one dimensional lattice type periodic structures need to be divided into two main classes:

(i) The Linear Chain,

(ii) The Cyclic Chain.

Each of these classes has its own peculiar characteristics which are dictated both by the geometry (boundary conditions) and the physics of the system. For example, in the linear chain, degenerate and therefore 'seemingly' unpredictable behavior under perturbations appears to occur only under very weak coupling conditions. Topologically, this behavior is equivalent to an unfolding of the m-fold (here m is the number of nominally identical subsystems which are weakly coupled) degeneracy: $(\lambda - \omega^2)^m = 0$. This corresponds to what Pierre has, in a series of papers, consistently referred to as a perturbation of the uncoupled system behavior. [9,10,11] This behavior does not exist under strong coupling conditions.

On the other hand for cyclic systems, even under very strong coupling conditions, extra degeneracy is induced by the cyclic symmetric nature of the system matrices. It is then well known that cyclic systems have several pairs of degenerate (coincident) eigenvalues which is distinct from the case of linear chains where no degeneracy or multiplicity of eigenvalues arises under strong coupling conditions. The <u>crucial observation is that because of the coincidence of</u>

eigenvalues, and the continuity of eigenvalues with respect to parameters, cyclic systems (to which bladed-disk assemblies belong) always operate in the neighborhood of the bifurcation set. For cyclic systems it is therefore of great interest to determine the relative influence of coupling and geometry in the subsequent degenerate system behavior. Since a tuned bladed-disk assembly has pairs of degenerate eigenvalues, the parameters corresponding to the tuned state are clearly a subset of the bifurcation set.

The number of degenerate pairs of eigenvalues as well as the effect of different types of perturbations depends on the nature of the symmetry. Some of the qualitative ramifications of the geometric symmetry can be studied using the theory of groups. The effects of symmetry preserving and symmetry breaking perturbations can be qualitatively studied using the ideas from perturbation of group action as well as the singularity theory for symmetric systems. While the results for universal unfolding of positive definite matrices and the behavior of eigenvalues for symmetry preserving perturbations are available [15], those for perturbations that destroy symmetry are not, and we will later present some examples displaying the interesting consequences of various types of perturbations. Finally, neither the group theory, nor the singularity theory, provide quantitative results such as formulas for the computation of the perturbed eigenvalues and eigenvectors as a function of the perturbation parameters. Only such information can provide the measures for eigenvalue loci veering and mode localization, and one possible tool for developing these expressions/results is the singular perturbation theory. Thus, tools or ideas from the disciplines of group theory, singularity theory, and singular perturbation theory, are all needed to make a strong headway in understanding the phenomenon of mode localization.

2. GROUP THEORY AND CYCLIC SYMMETRY

Although it had been observed that turned bladed-disk assemblies always have many pairs of degenerate eigenvalues, no theoretical justification for this phenomenon was available in the bladed-disk literature. Our first order of business in the investigation was therefore to obtain a formal explanation for this phenomenon. The coefficient matrices in the equations of motion of forced bladed-disk assemblies as well as the dynamic stiffness matrices are always banded circulant matrices [12]. These matrices have unique symmetry properties [13] which immediately indicate that group theory would be applicable. It turns out that the set of allowed symmetry operations in a bladed-disk assembly namely: rotations about a fixed axis, reflections about a fixed axis and vibrations about a reference point, can be captured by the operations of the Dihedral group D_n [14]. By purely formal arguments from group theory and standard results for the Dihedral group, we are able to show the number and order of degenerate eigenvalues which any finite order bladed-disk assembly can have. Furthermore by considering the irreducible representations to which the translational, rotational, and vibrational modes belong, along with the corresponding Hamiltonians, we can sufficiently study the effects of mild perturbations on these degenerate doublets. For example under a given parameter variation, the symmetry operations generate a new group which is necessarily a subgroup of the original group D_n. By comparing the properties of this new sub-group with those of the original group, we are able to determine if such a perturbation would lead to a splitting of any of the degenerate pairs of eigenvalues. It may then be possible to determine the minimum number of parameters which must be varied simultaneously in order for a certain number of degenerate eigenvalue pairs to be split at the same time. By now concentrating on those perturbations that lead to splitting of degenerate pairs we can more fully study the effects of these perturbations on the forced amplitude response of the assembly. These results are summarized in the paper in Appendix 1.

3. SINGULARITY THEORY AND CYCLIC SYMMETRY

It is known from singularity theory that the splitting of eigenvalues of a matrix can lead to rapid changes in the eigenvectors, which in turn can result in significant changes in the forced amplitude of response of the assembly to external aerodynamic loading. Since the group theory results indicate that bladed-disk assemblies could only have degenerate pairs of eigenvalues, the simplest essential properties of any finite order bladed-disk assembly are captured by the properties of an assembly of order 3. Note that we need a 3rd order assembly in order to inscribe a circle and hence obtain a cyclic system. A third order tuned assembly would thus have a degenerate pair of eigenvalues and an isolated eigenvalue. We may therefore study the influence of perturbations in the masses, ground springs and coupling springs on the dynamics of this system. From Arnold's results in singularity theory [16], it is self evident that under mild parameter variations interest should be concentrated not on the isolated eigenvalue but only on the subsequent behavior of the degenerate doublet. To understand its behavior it is necessary to study the behavior of any arbitrary doublet and the subsequent eigenloci as a function of parameters in a manner reminiscent of root loci behavior in classical control theory. The simplest doublet which contains the essential ingredients of the problem turns out to be the symmetric, coupled double pendulum shown in figure 1 in Appendix 2. The essense of this study was to discover the relation of the eigenloci to parameter variations and the corresponding eigenvectors. We had conjectured that:

(i) Uneven amplitudes of vibration in symmetric structural systems are caused by the sensitive dependence of system eigenvectors or parameters.

(ii) Sensitive dependence of eigenvectors on parameters gives rise under appropriate conditions to the mode localization phenomenon.

(iii) Rapid convergence-divergence (veering) of eigenvalues is a signature for the sensitive dependence of eigenvectors on parameters and hence of the possible existence of mode localization under appropriate conditions.

If the conjecture were to be true, we hoped to be able to obtain an estimate for the eigenvector sensitivity measure in an appropriate manner as well as an estimate for the eigenvector rotations resulting from any mild perturbation. If extreme imperfection sensitivity were present, it was expected that both measures would show a singularity which is an indication of imperfection sensitivity. Furthermore these were expected to occur at the parameter values of maximum curvature of the eigenloci. If the double pendulum were decoupled (no coupling spring) there then would exist two independent but equal vibration frequencies. By including very weak coupling between the masses we could study system behavior in the neighborhood of the erstwhile equal eigenvalues. The study of weakly coupled systems is very important since in practice the aim has always been towards use of rigid disks, in effect making the inter-blade coupling very weak indeed. By assuming that the imperfection parameter is a slight difference in the length of the two pendula, we could then study the behavior of the eigenvalues and eigenvectors of this simple symmetric system under slight changes in coupling and disorder.

4. QUANTITATIVE UNFOLDING OF THE MODAL SINGULARITIES BY SINGULAR PERTURBATION ANALYSIS

We may therefore write down the characteristic polynomial as a function of both the coupling parameter and the imperfection parameter. The characteristic polynomial in turn behaves identically to a potential function in singularity theory [17]. Thus the degenerate critical points of this function correspond to the repeated eigenvalues if any. By writing down the expressions for the eigenvalues as functions of the two parameters, the detailed behavior of the eigenloci in any neighborhood can be obtained. A regular expansion of these eigenvalues as a function of the two parameters breaks down (loses uniformity) in the neighborhood of the critical point (where the eigenvalues are coincident). By applying the techniques of singular perturbation analysis and appropriate stretching transformations it became possible to obtain the eigenloci expressions which were uniformly valid over the domain of definition of the small parameters and whose loci clearly indicated the veering phenomenon. The same technique was also applied to the eigenvector expressions as functions of the parameters. From these expansions, expressions were obtained both for the modal sensitivity measure and the eigenvector rotation measure under slight parameter variations. All the results obtained, confirmed the conjecture. The first part of these results are to appear in the Journal of Sound and Vibration while the second part involving the full eigenvector work has been submitted to the Journal of Sound and Vibration. These manuscript preprints are enclosed in Appendix 2. We can now claim that we understand fairly well the causes of localization phenomenon for simple doublets. We note however that the double pendulum analysis displayed the noted strange behavior only under very weak coupling conditions. Our work had shown earlier (see Appendix 4) that for bladed-disk assemblies uneven amplitudes of vibration and hence mode localization could occur even under very strong coupling conditions. We therefore had to discover under what conditions the double pendulum analysis remained valid also for the bladed-disk assembly. It turns out that the pendulum analysis remains valid for the bladed-disk, irrespective of the coupling strength. However by systematically reducing the coupling, more complicated singularities appear. This is because at very low coupling the bladed-disk behaves like a perturbation of a triple degeneracy which in general requires at least three parameters to unfold (completely analyse). On the other hand for the double pendulum, two parameters were enough to unfold the doublet degeneracy.

The lessons learned so far are thus that for the bladed-disks even under strong coupling conditions mode localization can occur. On the other hand for the double pendulum or linear chains in general, mode localization occurs only under weak coupling. This finding contradicts the current view in bladed-disk research [11], which holds that in both linear chains (coupled pendula) and cyclic chains (bladed-disk assemblies) mode localization only occurs under very weak coupling. What is however true for bladed-disk assemblies, is that under very weak coupling new singularity types (which do not exist under strong coupling) appear. We do not yet understand the full effects of these new singularity types. We however conjecture that they will further complicate the modal behavior of the assembly under aerodynamic loading. The key question we seek to answer presently is which of either symmetry breaking perturbations or coupling induced perturbations have more influence on the modal behavior of a bladed-disk assembly. Are there regimes where each has more influence than the other and if so, what is the transition region? If we could answer these questions then we could specify apriori the acceptable range of coupling so that design effort could be concentrated on symmetry breaking bifurcations and how to prevent their effects from being felt at the blade amplitudes. The singular perturbation analysis acts as an unfolding of the singularities involved, since by this methodology we are able to obtain detailed information on the modal behavior of the structure in the neighborhood of the singularities.

5. THE CONTROL PROBLEM

We have not carried out the control design component of the project as stated in the statement of work because it has only been in the last six to eight months that a thorough understanding of the structural dynamics has emerged. We however are clear on the work that needs to be done. The next stage of our work will involve classification of different perturbations with the corresponding amplitudes of vibration. The control problem in one possible approach is a structural redesign that deliberately breaks the symmetry by splitting the degenerate eigenvalues with only those perturbations that do not lead to amplification of vibration amplitudes. Provided the split eigenvalues are not in the neighborhood of the bifurcation set, all further slight perturbations would not be expected to display extreme imperfection sensitivity. Another alternative control methodology which we are presently considering is a regular adapative control scheme that seeks by means of active addition or subtraction of control masses and springs to restore symmetry whenever the symmetry breaking signature is observed. Under this scheme the degree of sensitivity and eigenvector rotation will determine the amount of modification called for and the location where to apply it. However this kind of scheme seems to us to be more appropriate for aerospace structural systems than to turbine rotor disks.

6. CONCLUSIONS

The results obtained from the study in the last two years have helped to clarify and unify several conflicting viewpoints within the bladed-disk research community. What is more significant is that it has led to a better understanding of the potentially very complicated dynamical structure which ensues when geometric (spatial) symmetry interacts with weak coupling in periodic structures. Without this understanding any attempts at either structural redesign or structural control of such systems would inevitably be frought with danger. We are currently continuing work on the forced response of cyclic systems with a view to a more complete mathematical characterization of the relationship between amplitudes of vibration, mode localization, and perturbation type. We are also generalizing the singular perturbation approach to linear chains and cyclic symmetric systems of any finite order. We believe that the development of the structural control schemes would be worthless without this full understanding.

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STATEMENT OF WORK

The principal aim of the proposed research is to carry out an in-depth mathematical and numerical investigation of the dynamics of mistuned cyclic systems, by use of some new and extremely powerful topological theory of dynamics, and to develop simple control schemes for preventing unacceptable vibration characteristics in such systems. To accomplish this task, we will:

(i) Identify the topological structure of nominally tuned bladed-disk assemblies, the order of the degeneracy in the natural frequencies, the minimum number of canonical parameters needed to unfold the degeneracy, and the classification of the bifurcation set in the parameter space.

(ii) Use the Jordan-Arnold canonical structure theory to completely characterize all the blade motion forms expected when a given nominally tuned system is generically mistuned.

(iii) Relate the canonical unfolding parameters to the disk assembly elements of mass, generalized damping and generalized stiffness; and hence determine which mistuning parameters or combinations thereof, govern the escalation of forced response amplitudes and/or unacceptable blade motions.

(iv) Employ the control methodologies of either entire eigenstructure assignment or quadratic optimization to deliberately mistune the blade assembly passively so that eigenvalue degeneracy under slight parameter variations are avoided and at the same time the parameter combinations which lead the assembly to unacceptable blade motions (the bifurcation set) are never allowed to occur.

(v) Carry out a thorough numerical simulation on typical nominal and perturbed bladed-disk assemblies to verify and validate the predictions of the new topological theory.

The above will set the stage for a controlled laboratory hardware experimental verification, which we hope to undertake in a follow-up project.

APPENDIX 1

On the Dynamics of Perturbed Symmetric Systems

by

G. Happawana

O.D.I. Nwokah

A. K. Bajaj

To be published in Proceedings of the 13th Biennial

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On the Dynamics of Perturbed Symmetric Systems

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December 1990

ABSTRACT

In this work, we consider the dynamics of linear mechanical systems possessing geometrical symmetry subject to differential or small parameter variations. The machinery of group theory including the irreducible group representations, and the consideration of representations to which the translational, rotational and vibrational modes belong, allow us to predict apriori, the number and the order of degenerate eigenvalues in the symmetric system. By considering the resultant Hamiltonians of the perturbed symmetric system, we show further the effects of the perturbations on the eigenvalues and their degeneracies. Since the vibration modes of systems with degenerate eigenvalues are known to display sensitive dependence on parameters, we may use these techniques to identify in principle the possibility of maximum vibration amplitudes and where they are likely to occur. Applications of these ideas include the mistuned turbine rotor bladed disk assemblies.

LIST OF SPECIAL SYMBOLS

$\Gamma^{C_{red}}(R)$	Reduced cartesian representation of a group element R.
$\Gamma^{t_{red}}(R)$	Reduced translational representation of a group element R.
$\Gamma^{r_{red}}$	Reduced Rotational representation.
$\Gamma^{v_{red}}$	Reduced Vibrational representation.
$\Gamma(R)$, $\Gamma^{i}(R)$	Matrix representations.
$\Gamma^{i}(R)$	i^{th} representation.
$\Gamma^{i}_{\mu\nu}(R)$	Matrix element of the μ^{th} row and the ν^{th} column of the matrix representing the group element R in the i^{th} representation.
$\Gamma^{i*}_{\mu\nu}(R)$	Complex conjugate of $\Gamma^{i}_{\mu\nu}(R)$.
$\chi^{i}(R)$	Character of a group element R in the i^{th} matrix representation.
a_i	Number of times $\Gamma^{i}(R)$ appears in the reducible representation.
$\tilde{e}$	Row vector.
M	Mass matrix.
K	Stiffness matrix.

1. INTRODUCTION

Eigenvalues and eigenvectors of a vibrating system are important for characterizing its dynamical response. The eigenvalues are related to natural frequencies whereas the eigenvectors correspond to special forms of displacements when vibrating at a natural frequency. Exact evaluation of the eigenvalues of higher order vibrating systems in general involves considerable effort and is time consuming. Most cyclic symmetric systems possess degenerate eigenvalues [1,2]. Systems with degenerate eigenvalues are expected to display severe sensitive dependence on parameters [3] that destroy the symmetry or degeneracy.

In the eigenvalue problem if there is any symmetry of the system, the application of group theory enables us to decide, at the outset, exactly the number of distinct eigenvalues together with their respective degrees and degeneracies.

By considering the symmetry operations of the physical system at the equilibrium points, the representing group can be formulated. Using group theoretical ideas, we can predict apriori the degeneracy of the eigenvalues. This is accomplished by the use of the irreducible representations of this group which is obtained by using the orthogonality theorem and the reduction formula [3]. Once the irreducible representations are known, we can find the translational, rotational and vibrational modes of the system. These results are well known in the physics literature on group theory but have not been used sufficiently effectively in the vibration community. The essential purpose of this work is to summarize some of these results and show some applications as they relate to the symmetric bladed disk assemblies.

In general, a reduced cartesian representation of a group element R, $\Gamma^{c\,red}(R)$, can be written as

$$\Gamma^{c_{red}}(R) = \Gamma^{t_{red}}(R) \oplus \Gamma^{r_{red}}(R) \oplus \Gamma^{v_{red}}(R),$$

where $\Gamma^{t_{red}}(R)$, $\Gamma^{r_{red}}(R)$, and $\Gamma^{v_{red}}(R)$ are translational, rotational and vibrational reduced cartesian representations of a group element R. In three dimensional vibrating systems, six of the normal modes belong to the zero frequency modes and correspond to pure translations and pure rotations. Since we are primarily interested in vibrational modes (non zero frequency) the zero frequency modes are not discussed further in this work.

Some of the symmetry of the physical system may be lost once the system is subjected to parameter perturbations. The perturbed system may, however, still possess some symmetry which may be considered to be a subgroup of the group characterizing the unperturbed or original system. Applying group theoretic ideas now to this subgroup we can predict the splitting of the degeneracy of eigenvalues. As a result we can see whether the degeneracy of some of the eigenvalues has or has not been removed.

Ideas similar to the ones proposed in this work were used by R. Perrin [4] in 1971 for a thin circular ring. In his paper group theoretical arguments were applied to a ring where perturbation was applied in the form of equal masses attached to the ring at the vertices of an inscribed regular n^{th} order polygon. Further, eigenfrequencies and eigenfunctions for the unperturbed ring were assumed to be known apriori. Knowledge of these degenerate pairs of eigenfunctions was used to find the characters of each irreducible representation of the corresponding D_n group for the perturbed system. In the present work, group theoretical techniques are developed without apriori assuming any knowledge of either the eigenfrequencies or the eigenfunctions for the circulant symmetric system. Also, parametrically perturbed cases were not discussed by Perrin. Parametric perturbations are important in turbine blade vibration problems where a slight perturbation can lead to loss of cyclic symmetry, which in turn can induce rogue blade failure under certain circumstances [5,6].

2. THE GROUP THEORETICAL CONCEPTS

We first define some standard terms from the literature on group theory. One of the standard references is the text by Hamermesh [1]. Following definitions and theorems are obtained from [2].

2.1 Definition 1. **Symmetry operations**: All the operations which leave a system configuration unchanged are called symmetry operations.

In physical terms, this refers to the movement of a system in such a way that it interchanges the positions of various particles of the system but results in the system looking exactly the same as before the symmetry operation. For instance some of the symmetry operations are defined as follows:

E: Identity. The system is not rotated at all or rotated by 2π about any axis.

C_n Rotation: This is an operation which effects rotation through an angle $2\pi/n$ about an axis, fixed in space, where n is an integer. In addition we can have C_n^k, which is C_n raised to the power k, that is, a rotation through an angle $2\pi k/n$ about the same axis. C_n^n is a rotation through an angle 2π and is the identity operation, since a rotation through 2π leaves the object unchanged. n is known as the multiplicity of the axis, and the latter is called on n fold axis. If n=2,3,... then, respectively, we get 2-fold, 3-fold... axes. If a system has more than one axis of symmetry then the axis with the highest value of n is called the principal axis.

Definition 2. **Group**: A set of elements {a,b,c..} is called a group G, if a multiplication rule is defined for any two elements so that the product ab has a definite meaning and the following four postulates are satisfied:

1. Closure: If a and b belong to the set, then ab also belongs to the set.
2. Associativity: a(bc) = (ab) c.
3. There exists the identity element e such that ae = ea = a for any a belonging to G.
4. There exists the inverse element, i.e., for each element a, there is a corresponding element b such that ab = ba = e. b is called the inverse element of a and is denoted by $b = a^{-1}$.

Definition 3. **D_n group**: This group concerns a system possessing one n-fold axis called the principal axis and n 2-fold axes symmetrically placed in a plane perpendicular to the principal axis. The n-fold axis provides the n elements of the cyclic group C_n. The group also contains one C_2 element provided by every perpendicular 2-fold axis where we do not count $C_2^2 = E$ because E only occurs once in a set of group elements and it has already appeared in the n elements of C_n. The group D_n therefore contains a total of 2n elements.

Definition 4. **Equivalent and reducible representations**: Two representations are said to be equivalent if the two matrices representing any element R of the group are related by the equation

$$\Gamma'(R) = T^{-1}\,\Gamma(R)\,T, \tag{1}$$

where T is any nonsingular square matrix (operator). However, if there does not exist any matrix T which transforms $\Gamma'(R)$ into $\Gamma(R)$, then $\Gamma'(R)$ and $\Gamma(R)$ are said to be inequivalent.

A reduced representation of a group element R, $\Gamma^{red}(R)$, is composed of two or more irreducible representations:

$$\Gamma^{red}(R) = \begin{bmatrix} \Gamma^i(R) & 0 \\ 0 & \Gamma^j(R) \end{bmatrix}. \tag{2}$$

We write this by using the symbol $\oplus$ and

$$\Gamma^{red}(R) = \Gamma^i(R) \oplus \Gamma^j(R). \tag{3}$$

Definition 5. $\chi^I(\mathbf{r})$, **Character of a group**: The character of a group element R in the i^{th} matrix representation of the group element is the trace (sum of diagonal terms) of the matrix.

2.2.1 Orthogonality Theorem: All the vectors formed by the inequivalent irreducible unitary representations are orthogonal to each other, or:

$$\sum_R \Gamma^{i*}_{\mu\nu}(R)\,\Gamma^{j}_{\mu'\nu'}(R) = \frac{g}{l_i}\,\delta_{ij}\delta_{\mu\mu'}\delta_{\nu\nu'},$$

where i and j denote the representation, μ and μ' denote rows of the matrix elements and v and

ν' denote the columns of the matrix elements, g is the order of the group and l_i is the dimensionality of the i^{th} representation. $\Gamma^i_{\mu\nu}$ (R) is the matrix element of the μ^{th} row and ν^{th} column of the matrix representing the group element R in the i^{th} representation, and * denote its complex conjugate equivalent to Γ^i (R)

2.2.2 Character table: The character table is formed by considering the characters of group elements. The character of a group element is important because the character is unaltered by a similarity transformation. On the other hand since the character of equivalent irreducible representations are identical a table of characters is a unique way to characterize a group. The general form of a character table is:

	N_1 c (1)	N_2 c (2) $\cdots\cdot$ N_r c (r)
Γ^1(R)	χ^1 (c (1))	χ^1 (c (2)) $\cdots$ χ^1 (c (r))
Γ^2(R)	χ^2 (c (1))	$\cdots\cdots\cdots\cdots$ χ^2 (c (r))
Γ^r(R)	χ^r (c (1))	$\cdots\cdots\cdots\cdots$ χ^r (c (r))

where

c(r): the nature of elements in the class,

N_i: number of elements in the class, and

Γ^i: i^{th} irreducible representation.

The character tables of the groups can be obtained by the application of the following rules:

(1) Number of inequivalent representations is equal to the number of classes.

(2) $\sum_{i=1}^{n} l_i^2 = g$, n- number of irreducible representations.

(3) $\sum_{R} \chi^{i*}(R)\, \chi^{j}(R) = g\, \delta_{ij}$, where $\chi^{i}(R)$ denote the character of a group element R in the ith matrix representations.

(4) $\sum_{i=1}^{n} \chi^{i*}(C_k)\, \chi^{i}(C_k) = \frac{g}{N_k}\, \delta_{kn}$.

In labeling the rows of the character table, the following standard notation is used.

(1) One-dimensional representations are labeled as A if the character of the elements C_n^k about the principal rotation axis are +1 for all k, and as B if the characters C_n^k are $(-1)^k$ for all k.

(2) If a group has more than one A or B representation they are given subscripts 1 and 2 according to whether the character is +1 or -1 in the column representing a rotation or improper rotation about an axis other than the Principal axis. For example, in the groups D_n a representation is given a subscript 1 if the character under C_2 about the axis is +1 and 2 if it is -1.

(3) Two dimensional representations are labeled E.

(4) Three dimensional representations are usually labeled T.

Finally, the reducible representations are used to obtain the irreducible representations by the applications of the formula

$$a_j = \frac{1}{g} \sum_{R} N_j\, \chi^{j*}(R)\, \chi^{red}(R)\,. \tag{4}$$

3. VIBRATIONS OF CYCLIC MECHANICAL SYSTEMS

Many vibrating systems possess sufficient symmetry to allow us to use group theory which reduces the amount of work involved in the calculations and also furnishes us with an insight into the nature of the vibrations. Consider n nominally identical masses connected via ground springs k_t and coupling springs k_c at the edges of an inscribed n^{th} order regular polygon in a circle of radius r. Rotational symmetry about one n-fold axis perpendicular to the plane of motion and n, 2-fold axes give 2n number of elements for the corresponding symmetry group. In fact, this is the dihedral group D_n. D_n has $(2 + \frac{n-1}{2})$ conjugate classes when n is odd and $(3 + \frac{n}{2})$ classes when n is even. Utilizing group character table construction rules [1,2], it can be shown that the number of possible degenerate eigen levels are:

n even

$$\text{i)} \quad \left.\begin{array}{c} \text{single degenerate levels} = 4 \\ \text{double degenerate levels} = \frac{n}{2} - 1 \end{array}\right\}, \tag{5}$$

n odd

$$\text{ii)} \quad \left.\begin{array}{c} \text{single degenerate levels} = 2 \\ \text{double degenerate levels} = \frac{n-1}{2} \end{array}\right\}. \tag{6}$$

Results (i) and (ii) imply that a cyclic symmetric system of this type, at worst, can have double degenerate vibrational modes for any finite n.

Once the system is subjected to a random parameter perturbation, some of the symmetry may be lost and consequently, we get a new group which is likely to be a subgroup of the original group. Equations (5) and (6) can be used to get a qualitative information about the new degenerate eigenvalues of the perturbed system. Since any higher order cyclic symmetric system of this type will have at most doubly degenerate eigenvalues it is sufficient to consider an example with n = 3.

Consider three normally identical masses connected via both, ground springs k_t and coupling springs k_c, at the edges of an inscribed isosceles triangle in a circle of radius r. We wish to study the number and degeneracy of the eigenvalues of such systems under random differential perturbations in the elements $\{k_t, k_c, m\}$.

As defined in Section 2, a symmetry operation is one which leaves the undistorted system indistinguishable from its previous orientation. Such an operation interchanges equivalent masses. However, in the vibrational state, the system is in a distorted configuration and, when the symmetry operation is performed on the distorted mass the effect is the same as that obtained by interchanging displacement vectors amongst equivalent masses. Therefore we can define the action of a symmetry operation for each mass in a distorted system to be a displacement through vector X_i from its equilibrium. When a symmetry operation is applied we can assume that the mass positions remain invariant. Furthermore the symmetry operation can have no effect on the potential or kinetic energy of the system, or even the angles between the connections. Consequently the quadratic forms of the kinetic energy T and the potential energy V remain invariant under the action of the group transformations. Group theory can thus be used to determine and classify the normal modes of the vibrating system.

We begin with the 3N dimensional representation of the group of symmetry operations of the undistorted system. By reducing this representation using the character table for the corresponding group, (and reduction of reducible representations) we can determine the irreducible representations to which the 3N translational, rotational and vibrational modes belong. Also we can immediately find the degeneracy of each normal mode. In addition, by considering the symmetrized basis S, we can bring the mass matrix M and the stiffness matrix K into block form and thus greatly simplifying the solution of the characteristic or the frequency equation.

We apply these group theoretic techniques to the problem shown in Figure 1 [4]. This system belongs to the group D_3 which contains the symmetry operations E, C_3^1, C_3^2, C_2^a, C_2^b, and C_2^c. Now by applying these operations to the nine cartesian coordinates $x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3$ we obtain their nine dimensional reducible representations. This can be accomplished by finding the corresponding matrix representation, and using the equation

$$X' = \Gamma^c(R)X\,, \tag{7}$$

where R is the appropriate symmetry operation, and X and X′ denote the nine dimensional vectors representing the cartesian coordinates in original and transformed planes respectively. For example, under the operation of C_3^1 the system configuration in Figure 1 is transformed to the configuration in Figure 2. This also clearly indicates the manner in which the coordinates undergo rotation. In Figure 2, the axes z_1, z_2 and z_3 are pointing out of the plane of the paper. The new and the old coordinates are related by the relations

$$(x_1, y_1, z_1) \rightarrow (x'_1, y'_1, z'_1) \equiv (-x_2 \sin 30 - y_2 \cos 30,\ x_2 \cos 30 - y_2 \sin 30,\ z_2)\,, \tag{8}$$

$$(x_2, y_2, z_2) \rightarrow (x'_2, y'_2, z'_2) \equiv (-x_3 \sin 30 - y_3 \cos 30,\ x_3 \cos 30 - y_2 \sin 30,\ z_3)\,, \tag{9}$$

$$(x_3, y_3, z_3) \rightarrow (x'_3, y'_3, z'_3) \equiv (-x_3 \sin 30 - y_3 \cos 30,\ x_3 \cos 30 - y_2 \sin 30,\ z_3)\,. \tag{10}$$

In matrix form, equations (8) - (10) are represented as

$$
\begin{Bmatrix} x_1' \\ x_2' \\ x_3' \\ y_1' \\ y_2' \\ y_3' \\ z_1' \\ z_2' \\ z_3' \end{Bmatrix}
=
\begin{bmatrix}
0 & -1/2 & 0 & 0 & -\sqrt{3}/2 & 0 & 0 & 0 & 0 \\
0 & 0 & -1/2 & 0 & 0 & -\sqrt{3}/2 & 0 & 0 & 0 \\
-1/2 & 0 & 0 & -\sqrt{3}/2 & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{3}/2 & 0 & 0 & -1/2 & 0 & 0 & 0 & 0 \\
0 & 0 & \sqrt{3}/2 & 0 & 0 & -1/2 & 0 & 0 & 0 \\
-\sqrt{3}/2 & 0 & 0 & -1/2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \\ z_1 \\ z_2 \\ z_3 \end{Bmatrix}, \tag{11}
$$

where the coefficient matrix is the matrix representation of the group element C_3^1. The character (trace) of the matrix is then

$$\chi^c\,(C_3^1) = 0\,. \tag{12}$$

Since C_3^2 and C_3^1 belong to the same class, $\chi^c\,(C_3^2) = 0$. It is clear that $\chi^c\,(E) = 9$. In a similar manner

$$
\begin{aligned}
\chi^c\,(C_2^a) &= -\,1\,, \\
\chi^c\,(C_2^b) &= -\,1\,, \\
\chi^c\,(C_2^c) &= -\,1\,.
\end{aligned} \tag{13}
$$

Now using the character table for D_3 (Table 1) and equation (4), we can determine the irreducible components of $\Gamma^c\,(R)$ as:

Table 1

	D_3	E	$2\,C_3$	$3\,C_2$		
$\Gamma^{(1)}$	A_1	1	1	1		
$\Gamma^{(2)}$	A_2	1	1	-1	T_z	R_z
$\Gamma^{(3)}$	E	2	-1	0	T_x, T_y	R_x, R_y

Here, A_1, A_2 represent one dimensional representations, E represents two dimensional representations, and T_x, T_y, T_z and R_x, R_y, R_z represent unit translational and rotational vectors respectively. Consequently

$$a_{A_1} = \frac{1}{6}\,(9 \times 1 + 2\,(0 \times 1) + 3\,(1 \times -1)) = 1\ ,$$

$$a_{A_2} = \frac{1}{6}\,(9 \times 1 + 2\,(0 \times 1) + 3\,(-1 \times -1)) = 2\ ,$$

$$a_{A_3} = \frac{1}{6}\,(9 \times 2 + 2\,(0 \times -1) + 3\,(1 \times 0)) = 3\ .$$

Hence $\Gamma^{C_{red}}(R) = A_1 \oplus 2A_2 \oplus 3E$. Since the system has three masses, $N = 3$ and there are 9 degrees-of-freedom for the system. Since $3N - 6 = 3$, we have only three vibrational modes, the other six corresponding to zero frequency modes and to pure rotations and pure translations. This can be seen from the character table,

$$\Gamma^{t}(R) = A_2 \oplus E\ ,$$

$$\Gamma^{r}(R) = A_2 \oplus E.$$

Since we are primarily interested in the vibrational modes of the system and

$$\Gamma^{C_{red}}(R) = A_1 \oplus 2A_2 \oplus 3E\ ,$$

$$\Gamma^{t}(R) \oplus \Gamma^{r}(R) = 2A_2 \oplus 2E,$$

we get $\Gamma^{V_{red}}(R) = A_1 \oplus E$.

Since A_1 is one dimensional, the vibrating system has one non degenerate eigenvalue and since E is 2 dimensional there is a one degenerate eigenvalue. These results are consistent with the exact eigenvalues given in appendix A.

Using group theory arguments to predict the number of degenerate eigenvalues becomes very useful as the order of the system increases since the exact calculation of eigenvalues for such high order systems becomes increasingly burdensome.

4. THE PERTURBED SYSTEM

Suppose that the Hamiltonian of the unperturbed system is H_0. Then H_0 is invariant under its symmetry group G. Suppose further that the system is subjected to a perturbation with Hamiltonian V. The perturbed Hamiltonian $H = H_0 + V$, will then have a symmetry group which is necessarily a subgroup of G. Two possible cases arise.

CASE I

If the perturbation V has symmetry at least as great as H_0, the group G will still be the symmetry group of the total Hamiltonian H. In this case the possible types of eigenvalues will be unchanged by the perturbation. In fact no splitting of degenerate levels occurs.

CASE II

If the perturbation V has symmetry lower than H_0, the total Hamiltonian H will have a symmetry group G^1 which is a subgroup of G. This subgroup G^1 is invariant under the perturbation. Because of the perturbation, some of the degenerate eigenvalues may split. This can be explained by using group representation theory.

For a given representation D(G) of the group G, we now obtain the invariant subgroup G^1. Even if D(G) is an irreducible representation of G, the representations of G^1 which we derive in this way may be reducible. In other words, even though we cannot find a subset of the basis vectors of D(G) which is invariant under all transformations of the group G, we may be able to find a subspace which is invariant under all transformations belonging to the eigenvalue λ from a basis of an irreducible representation of G. This representation may be reducible for the subgroup G^1. The perturbation V will then split the level.

We now apply the above ideas and show the appropriate methodology in the context of the three mass system.

4.1 Ground Spring Perturbed System

First we consider the situation when one of the ground springs is perturbed. The unperturbed system can be represented by the group $D_3 = \{E, C_3, C_3^2, C_a, C_b, C_c\}$. Once the system is subjected to a ground spring perturbation, for this particular system $G^1 = \{E, C_a\}$ is the invariant subgroup.

The character table for $G^1 = \{E, C_a\}$, is as follows:

	E	C_a
A′	1	1
A″	1	-1 .

Considering the part of the character table of D_3 which refers to the operations of the subgroup $G^1 = \{E, C_a\}$, we have

	E	C_a
E	2	0 .

Utilizing (4), the irreducible components are then given by

$$a_{A'} = \frac{1}{2}\,(1 \times 2 \times 1 + 1 \times 0 \times 1) = 1\ ,$$

$$a_{A''} = \frac{1}{2}\,(1 \times 2 \times 1 + 1 \times 0 - 1) = 1\ .$$

Thus, the doubly degenerate E level of the unperturbed system splits into single levels A′ and A″ of G^1 under the ground spring perturbation. As a result, degenerate eigenvalue of the perturbed system separates. Hence, for this particular system we get three distinct eigenvalues. Coupling spring perturbation leads to a case where there is no invariant subgroup left and consequently, group theoretical arguments do not work for this particular situation. We conjecture that this is indicative of those cases where perturbations do not lead to radical changes in the eigenvector directions.

4.2 The Mass Perturbed System

A system consisting of three particles, two with mass m and the other with mass M, is illustrated in Figure 3. Considering rotational symmetry of the system, we can see from Figure 3 that this system belongs to the group D_2

$$D_2 = \{E,\ C_2\}\ .$$

The character of each of the elements of the group can be obtained from the reducible representation whose matrix representation is obtained by the use of the coordinate transformation

$$X^1 = \Gamma^c\,(R)\,X.$$

Performing these operations, and following the steps along the lines of work in section 3, we can show that the reducible characters of each element are

$$\chi^c\,(C_2) = -1\ ,$$

$$\chi^c\,(E) = 9.$$

Using the character table of D_2 and (4), the irreducible components of the reducible representations can be then determined as

D_2	E	C_2		
A	1	1	z	R_z
B	1	-1	x,y	R_x, R_y

$$a_A = \frac{1}{2}\left[9 \times 1 - 1 \times 1\right] - 4\,,$$

$$a_B = \frac{1}{2}\left[9 \times 1 + -1 \times - 1\right] = 5.$$

Therefore,

$$\Gamma^{C_{red}}\,(R) = 4A \oplus 5B\,. \tag{14}$$

In general, $\Gamma^{C_{red}}\,(R) = \Gamma^{t_{red}}\,(R) \oplus \Gamma^{r_{red}}\,(R) \oplus \Gamma^{V_{red}}\,(R)$.

By placing a coordinate system XYZ at the center of mass, translations and rotations can be represented as shown in Figure 4. The representations for C_2 and E are given by

$$\chi^t\,(C_2) = -1 \quad , \quad \chi^t\,(E) = 3\,,$$

$$\chi^r\,(C_2) = -1 \quad , \quad \chi^r\,(E) = 3\,.$$

By the application of the a_i equations (4), we get

(i) Translation

$$a_A^t = \frac{1}{2}\,[1 \times 3 - 1 \times 1] = 1\ ,$$

$$a_B^t = \frac{1}{2}\,[1 \times 3 - 1 \times -1] = 2\ .$$

(ii) Rotation

$$a_A^r = \frac{1}{2}\,[1 \times 3 - 1 \times 1] = 1\ ,$$

$$a_B^r = \frac{1}{2}\,[1 \times 3 - 1 \times -1] = 2\ .$$

Therefore,

$$\Gamma^{t_{red}}\,(R) = A \oplus 2\,B\ , \tag{15}$$

$$\Gamma^{r_{red}}\,(R) = A \oplus 2\,B\ . \tag{16}$$

Hence from equations (14)-(16), we get the result that

$$\Gamma^{v_{red}}\,(R) = 2\,A \oplus B\ . \tag{17}$$

This shows that there are three vibrational modes and each eigen level is non-degenerate since A and B are one dimensional representations.

5. SUMMARY AND CONCLUSIONS

This work uses results from group theory and applies it to perturbed cyclic symmetric vibratory systems. It is shown that:

a. The number and order of degenerate eigenvalues in a symmetric system can be predicted apriori by using group theory without explicitly determining the eigenvalues.

b. Cyclic symmetric vibrating systems possess degenerate eigenvalues for n > 2. For strong coupling, these eigenvalues occur in doubly degenerate pairs and in single nondegenerate levels.

c. Random parameter perturbations may partially or totally destroy the symmetry of the system. Accordingly these perturbations lift some of the degeneracy of eigenvalues. As a result, eigenvalue loci veering [7] occurs when the parameters are continuously varied. This may also lead to a mode localization or rapid variation in the eigenfunctions [7].

ACKNOWLEDGEMENT

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Appendix A

Equations of motion for the system in Figure 2 can be written in the form:

$$M\ddot{x} + Kx = 0 \quad , \quad \text{where}$$

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \quad , \quad K = \begin{bmatrix} a & -k_c & -k_c \\ -k_c & a & -k_c \\ -k_c & -k_c & a \end{bmatrix} ,$$

and

$$a = 2k_c + \frac{k_t}{r^2} .$$

The eigenvalues are determined by

$$\det \mid K - \omega^2 M \mid = 0 .$$

Therefore,

$$\omega_1^2 = \frac{k_t}{mr^2} + \frac{3\,k_c}{m} \quad , \quad \omega_2^2 = \frac{k_t}{mr^2} + \frac{3\,k_c}{m} , \quad \text{and}$$

$$\omega_3^2 = \frac{k_t}{mr^2} .$$

This shows that the cyclic symmetric system has a double degenerate eigenvalue and a single nondegenerate eigenvalue.

LIST OF FIGURES

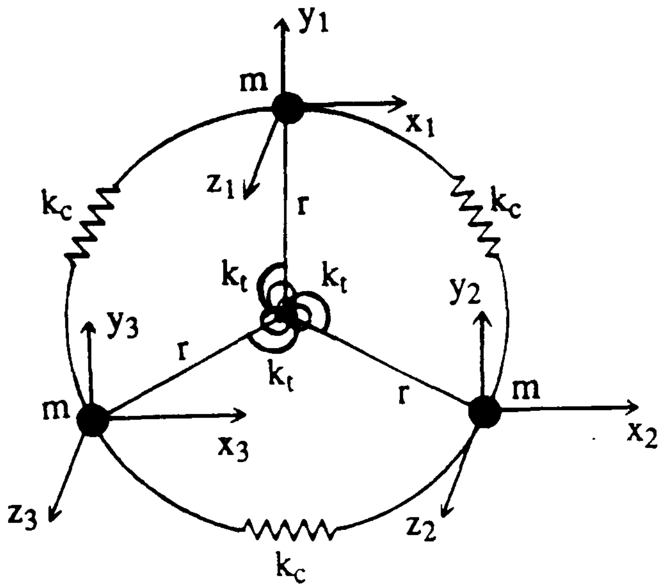

Figure 1. Model representing the unperturbed three bladed disk assembly in its identity orientation.

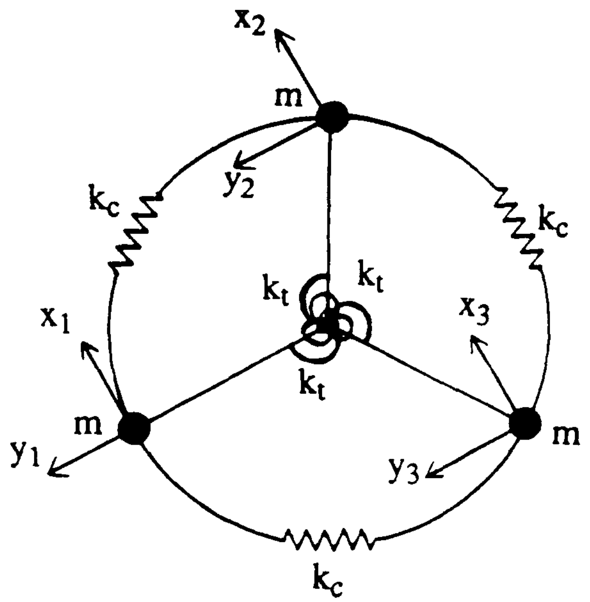

Figure 2. Model of figure 1 counter/clockwise rotated by $2\pi/3$ radians about the center of the system, showing change in cartesian coordinate axes.

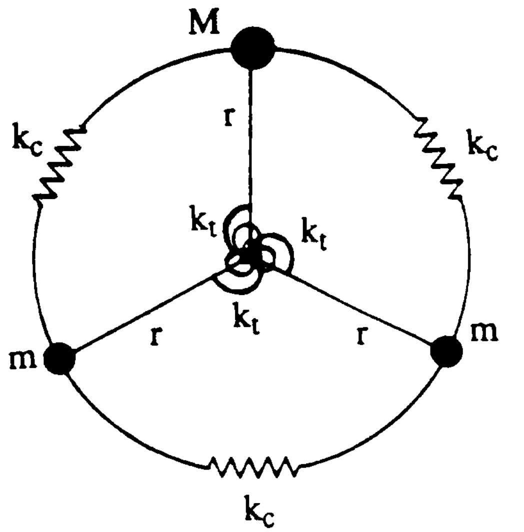

Figure 3. Mass perturbed system in its identity orientation.

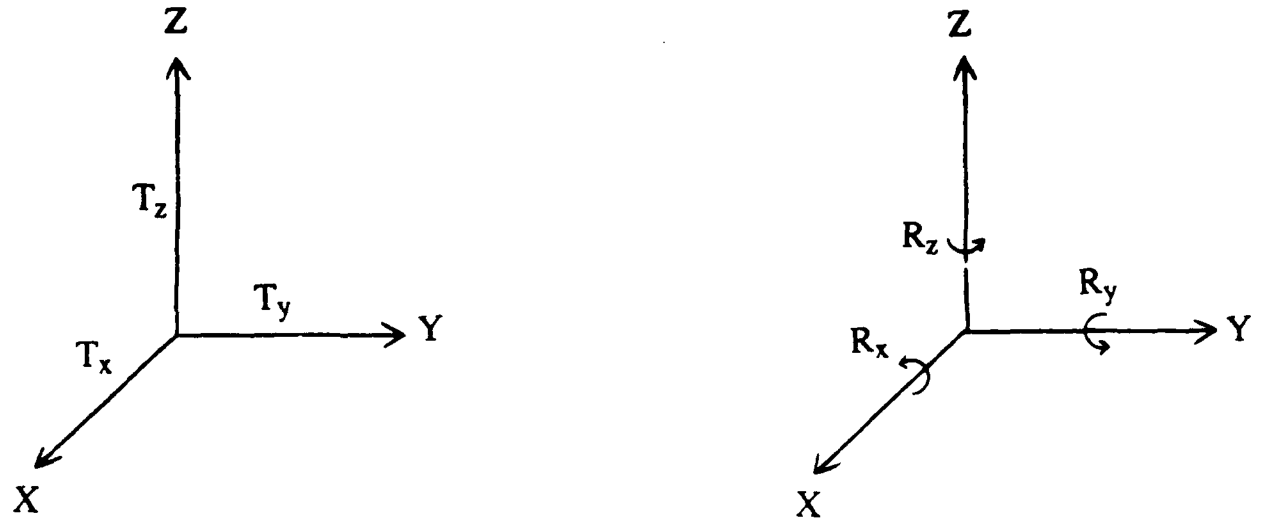

Figure 4. The representation of the three translations and the three rotations in an XYZ coordinate system.

APPENDIX 2

1. A Singular Perturbation Perspective on Mode Localization

   - G.S. Happawana, A. K. Bajaj, O.D.I. Nwoakah

   To appear in Journal of Sound and Vibration

2. A Singular Perturbation Analysis of Eigenvalue Veering and Mode Localization in Perturbed Linear Chain and Cyclic Systems

   G. S. Happawana, A. K. Bajaj, O.D.I. Nwokah

   Submitted to: Journal of Sound and Vibration

A Singular Perturbation Perspective on Mode Localization

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In recent years there has been tremendous interest in the vibrations and structural dynamics community in the phenomenon of mode localization. This interest stems from the recognition that large systems composed of nominally identical subsystems inevitably involve minor deviations from the idealized structures and these disorders or perturbations can, under appropriate conditions, cause disproportionately large deviations from the predicted behavior in the nominal or idealized system modes. Important technical applications of these include mistuned bladed disk assemblies [1] and large space structures [2].

It is well understood by now that the presence of small irregularities in nearly periodic structures may inhibit the propagation of vibration and localize the vibration modes. Depending on the magnitude of perturbations (disorder) and on the strength of internal coupling between the subsystems, the mode shapes may undergo dramatic changes to become strongly localized when small perturbations are introduced, thereby confining the energy associated with a given mode to a small geometric region. This phenomenon is referred to as mode localization. Pierre [3] showed that strong mode localization and eigenvalue curve veering, are two manifestations of the same phenomenon. Therefore, the investigation of the curves of the eigenvalues or natural frequencies in the neighborhood of the ordered state is sufficient for determining the occurrence of strong mode localization. That eigenloci veering phenomenon can occur in disordered structures under certain conditions has been explained qualitatively using geometric arguments in [4]; where it was also hinted that quantitative results should be obtainable by the use of singular perturbation analysis.

For mistuned linear chains, Pierre [3] and, Pierre and Dowell [5] showed that the straight forward expansion in terms of mistuning parameters breakdown in the case of weak coupling. This arises because the idealized system that is being perturbed has natural frequencies with multiplicity > 1. They then developed a so-called "modified" perturbation technique which provided a good approximation to the exact eigenfrequencies and showed good agreement with experimental results.

In the present note we show that the singularity causing the breakdown of the straight forward expansion can be analyzed by the well developed singular perturbation techniques [6] and an appropriate asymptotic expansion for the eigenfrequencies can be constructed which provides a correct qualitative and good quantitative approximation. In order to explain the ideas and to keep the algebraic manipulations to a minimum, the attention is focused on the now standard example [3] of the coupled penduli shown in Figure 1.

The basic idea of the technique is the following: by applying the regular perturbation technique to the characteristic equation $F(\lambda, \varepsilon, \delta) = 0$, of the system, we can obtain algebraic expressions for the natural frequencies (eigenvalues) as a power series in the small parameter or perturbation (say δ). The coefficients of the power series are dependent on the second parameter ε and these expansions are valid for all values of ε so long as no singularities arise. Singularities occur for values of ε where the eigenfrequencies lose their smoothness and it is said that the expansion is not uniformly valid for all ε. Away from the singular parameter (ε) values, the straightforward expansions are good approximations and are called the "outer expansions". The neighborhood of the singular parameter point is then stretched or rescaled in terms of a new parameter so as to remove the singularity. The expansions in terms of the new parameter is valid only in the neighborhood of the singular point and is called the "inner expansion". The inner and the outer solutions can be matched where their domains of validity overlap and then a composite expansion can be constructed which is valid uniformly throughout the function domain for all values of the parameter ε.

Consider the system of two weakly coupled penduli system as shown in Figure 1. The two important parameters are the dimensionless coupling between pendulums $R^2 = (k/m)/(g/l)$, and the dimensionless length change Δl. The corresponding eigenvalue problem generated by the above system is given by:

$$\begin{bmatrix} 1+R^2 & -R^2 \\ -R^2 & R^2+(1+\Delta l)^{-1} \end{bmatrix} \Phi = \lambda \Phi \tag{1.1}$$

where $R^2 = \omega_k^2/\omega_g^2$, $\omega_k^2 = k/m$, $\omega_g^2 = g/l$.

This eigenvalue problem results in the following characteristic equation:

$$F(\lambda,\varepsilon,\delta) = \lambda^2 - (1+2\delta+\frac{1}{1+\varepsilon})\lambda + \frac{1+\delta}{1+\varepsilon} + \delta = 0 , \tag{1.2}$$

where $\Delta l = \varepsilon$ and $R^2 = \delta$.

We can express the solutions to (1.2) as regular functions of the parameters Δl, δ as follows:

$$\lambda_1(\Delta l) = 1 + \delta + \left[1 + \frac{1}{\Delta l}\right] \delta^2 + O(\delta^3) \equiv \lambda_1^{\pm} (\Delta l) , \tag{1.3}$$

$$\lambda_2(\Delta l) = \frac{1}{1+\Delta l} + \delta - \left[1 + \frac{1}{\Delta l}\right] \delta^2 + O(\delta^3) \equiv \lambda_2^{\pm} (\Delta l) . \tag{1.4}$$

The expressions (1.3), (1.4) are the regular expansions of the eigenvalue problem for small coupling. When $\Delta l \to 0$, λ_1 and λ_2 become unbounded and the continuity of the eigenvalues with respect to the perturbation Δl breaks down, as shown in Figure 2. Each eigencurve has two branches, one valid for $\Delta l > 0$ and the other for $\Delta l < 0$. These branches are indicated by the superscripts '+' and '-' which correspond to $\Delta l > 0$ and $\Delta l < 0$, respectively. Note that, since Δl and $R^2 = \delta$ have been treated as two independent parameters, we have no control over expressions (1.3) and (1.4) in the limiting process when $\Delta l \to 0$ and $\delta \to 0$. By forming the "inner expansion" however, we can find an exact relation between these two parameters by taking into consideration the nature of the singularity. Then Δl and δ become dependent parameters. Asymptotically matching the inner and the outer expansions, then gives the composite expansions which are valid throughout the region of interest.

For the inner expansion, we assume that the physical parameters $R^2 = \delta$ and $\Delta l = \varepsilon$ are related (dependent) by a set of mathematical parameters: ξ_1, ξ_2, ξ_3 . . and μ by a "stretching" transformation of the form:

$$\varepsilon = \varepsilon_0 + \xi\mu^a + \sum_{j=2}^{\infty} \xi_j (\mu^a)^j \, , \tag{1.5}$$

where μ is a new small parameter that is defined by:

$$\delta(\mu) = (\mathrm{sgn}\,\delta)\mu^b \, . \tag{1.6}$$

The positive constants a and b are to be determined by the nature of the singularities of $F(\lambda, \varepsilon, 0)$ near $\varepsilon = \varepsilon_0$, where ε_0 is the singular point of interest. Let the dependent variable $z(\mu) = \lambda(\varepsilon(\mu), \delta(\mu))$ be written as the expansion

$$z = \sum_{j=0}^{\infty} z_j \mu^j \, . \tag{1.7}$$

Note that for the pendulum problem $F(\lambda, \varepsilon, 0) = 0$ has a singular point at $\varepsilon = \varepsilon_0 = 0$. The expansions (1.7) are called the "inner expansions" and the z_j's are called the "inner coefficients". Substituting (1.5), (1.6) and (1.7) into (1.2), and by simplifying the inner expansions with a=b=2 one obtains the following solutions:

$$z_1 = 1 + \left[\frac{2-\xi+\sqrt{\xi^2+4}}{2}\right]\mu^2 + \frac{\xi^2}{2}\left[1 - \frac{\xi}{\sqrt{\xi^2+4}}\right]\mu^4 + O(\mu^5)\,,$$

$$z_2 = 1 + \left[\frac{2-\xi-\sqrt{\xi^2+4}}{2}\right]\mu^2 + \frac{\xi^2}{2}\left[1 + \frac{\xi}{\sqrt{\xi^2+4}}\right]\mu^4 + O(\mu^5)\,, \tag{1.9}$$

$$\Delta l = \varepsilon = \xi\mu^2\,, \tag{1.10}$$

and

$$R^2 = \delta = \mu^2\,. \tag{1.11}$$

Keeping μ fixed and taking the limit $|\xi| \to \infty$, it is easy to see that z_1 matches asymptotically with λ_1^+ for $\xi \to \infty$ and with λ_2^- for $\xi \to -\infty$. Similarly the inner solution z_2 matches asymptotically ($|\xi| \to \infty$) with λ_1^- and λ_2^+. Now combining the inner and the outer expansions appropriately, we get the composite expansions:

$$\lambda_1 = \left[1 + R^2 + (1+\frac{1}{\Delta l})R^4\right](1-u(\Delta l)) + \left[1 + \frac{2R^2 - \Delta l - R^2\sqrt{(\Delta l/R^2)^2+4}}{2}\right.$$

$$\left. + \frac{\Delta l^2}{2}\left\{1 + \frac{\Delta l}{R^2\sqrt{(\Delta l/R^2)^2+4}}\right\}\right]$$

$$+ \left[\frac{1}{1+\Delta l} + R^2 - (1+\frac{1}{\Delta l})R^4\right]u(\Delta l) - (1-u(\Delta l))\left[1 + R^2 + (1+\frac{1}{\Delta l})R^4\right]$$

$$- u(\Delta l)\left[1 - \Delta l + \Delta l^2 + R^2 - (1+\frac{1}{\Delta l})R^4\right] + O(R^6)\,, \tag{1.12}$$

$$\lambda_2 = \left[\frac{1}{1+\Delta l} + R^2 - (1 + \frac{1}{\Delta l})R^4\right](1 - u(\Delta l)) + \left[1 + \frac{2R^2 - \Delta l + R^2\sqrt{(\Delta l/R^2)^2 + 4}}{2}\right.$$

$$\left. + \frac{\Delta l^2}{2}\left\{1 - \frac{\Delta l}{R^2\sqrt{(\Delta l/R^2)^2 + 4}}\right\}\right]$$

$$+ \left[1 + R^2 + (1 + \frac{1}{\Delta l})R^4\right]u(\Delta l) - \left[1 - \Delta l + \Delta l^2 + R^2 - (1 + \frac{1}{\Delta l})R^4\right](1 - u(\Delta l))$$

$$- [1 + R^2 + (1 + \frac{1}{\Delta l})R^4]u(\Delta l) + O(R^6) \,,$$

where

$$u(\Delta l) = \begin{cases} 1 \,, & \Delta l \geq 0 \\ 0 \,, & \Delta l < 0 \end{cases}$$

The plots of eigenfrequencies λ_1, λ_2 versus Δl are given in Figure 3 for both the exact solutions and the solutions obtained above by the singular perturbation technique. These are in excellent agreement. Thus, the singular perturbation technique leads to qualitatively correct asymptotic approximations that are often close to true solutions and can be used as a mathematical tool to generate quantitatively accurate solutions for a wide variety of linear and nonlinear structural dynamics problems. The methodology is general and systematic and when combined with elementary singularity theory, should provide a powerful technique to study the mode localization phenomenon in any finite order linear or cyclic dynamic chain.

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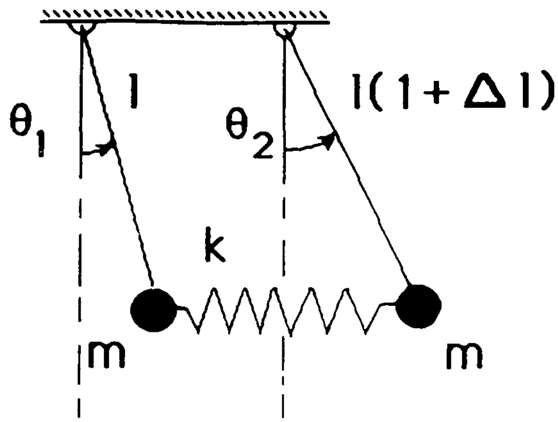

Figure 1. Two coupled oscillators.

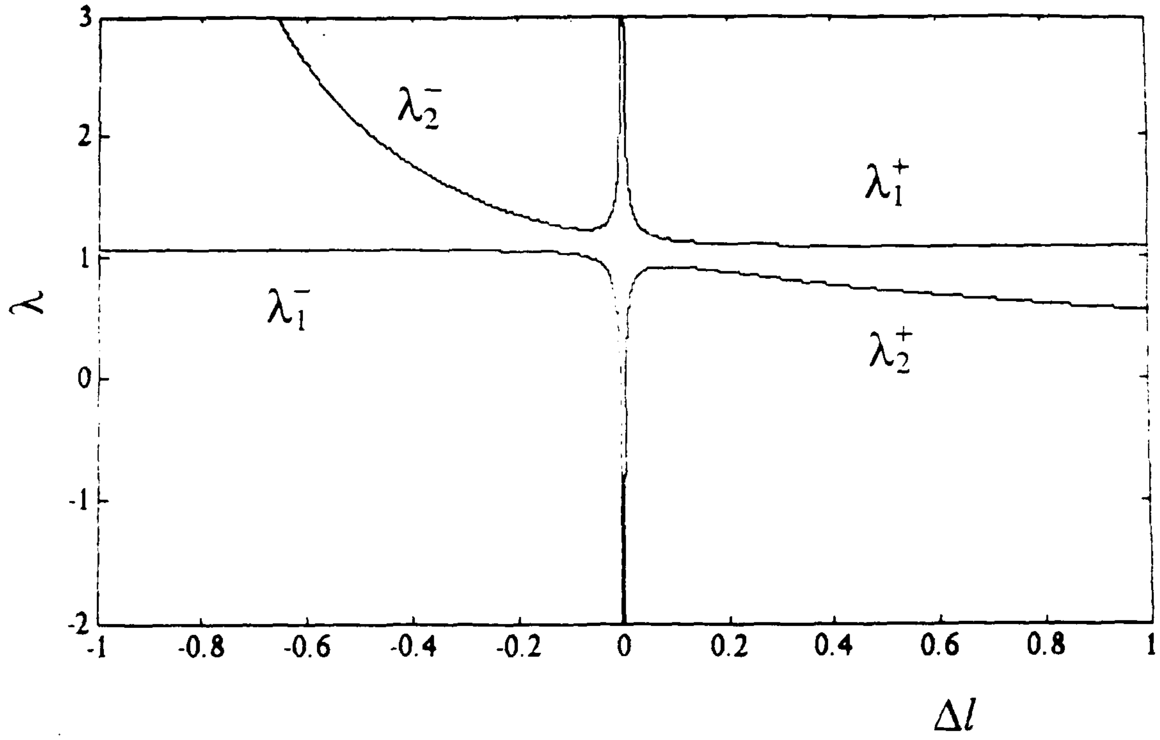

Figure 2. Outer expansions for eigenvalues indicating the region of singular behavior: R = 0.075.

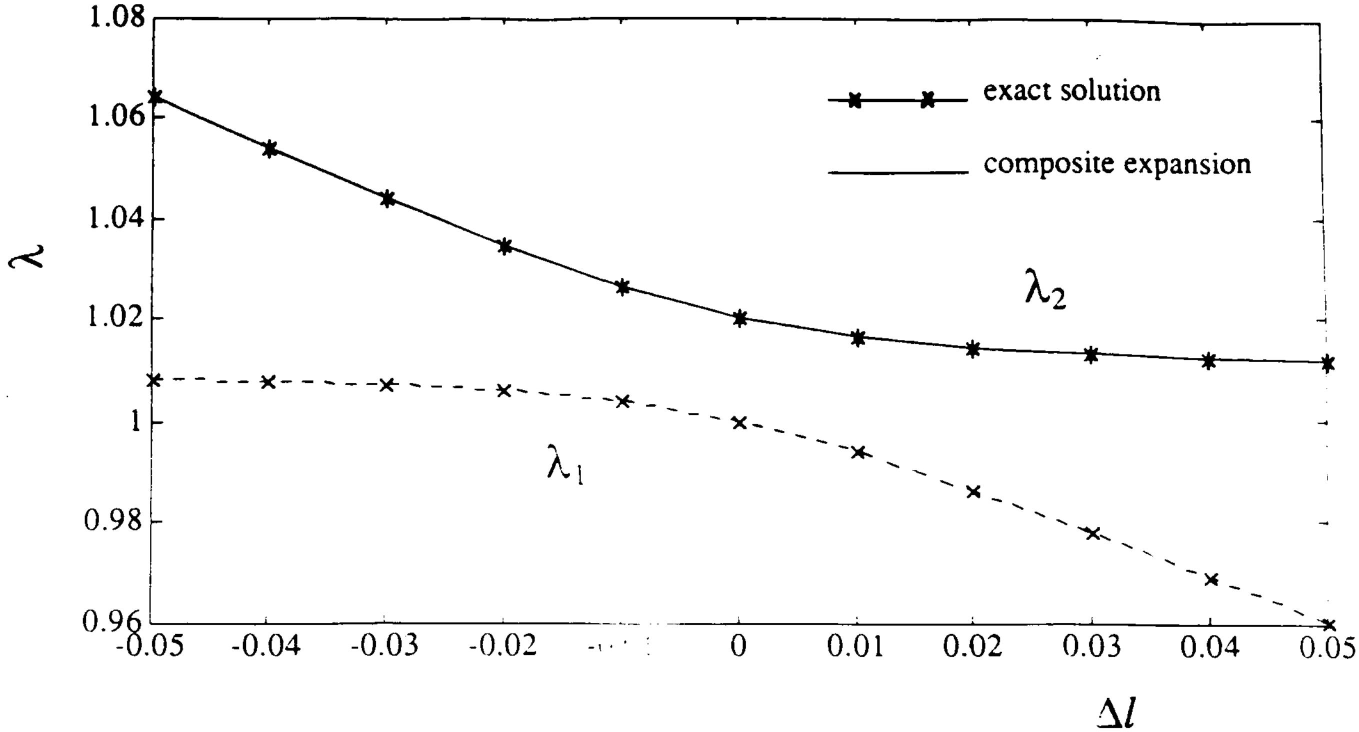

Figure 3. Comparison of eigenvalue curves from the exact solutions with those from the composite expansions; R = 0.1.

A Singular Perturbation Analysis of Eigenvalue Veering and Mode Localization in Perturbed Linear Chain and Cyclic Systems

by

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ABSTRACT

An investigation of the eigenvalue loci veering and mode localization phenomenon is presented for mistuned structural systems. Examples from both, the weakly coupled uniaxial component systems and the cyclic symmetric systems, are considered. The analysis is based on the singular perturbation techniques. It is shown that uniform asymptotic expansions for the eigenvalues and eigenvectors can be constructed in terms of the mistuning parameters and these solutions are in excellent agreement with the exact solutions. The asymptotic expansions are then used to clearly show how singular behavior in the eigenfunctions or modeshapes leads to mode localization.

1. INTRODUCTION

In recent years there has been tremendous interest in the vibrations and structural dynamics community in the phenomenon of mode localization. This interest stems from the recognition that large systems composed of nominally identical subsystems inevitably involve minor deviations from the idealized structures and these disorders or perturbations can, under appropriate conditions, cause unexpectedly large deviations from the predicted behavior in the nominal or idealized system modes. Important technical applications where these problems arise include mistuned bladed disk assemblies [1,2] and large space structures [3,4].

It is well understood by now that the presence of small irregularities in nearly periodic structures may inhibit the propagation of vibration and localize the vibration modes. Depending on the magnitude of perturbations (disorder) in the individual components and the strength of internal coupling between the subsystems, the mode shapes may undergo dramatic changes and become strongly localized when small perturbations are introduced, thereby confining the energy associated with a given mode to a small geometric region. This phenomenon is referred to as mode localization. Pierre [5] suggested that strong mode localization and eigenvalue curve veering are two manifestations of the same phenomenon. Therefore, the investigation of the curves of the eigenvalues or natural frequencies in the neighborhood of the ordered state is sufficient for determining the occurrence of strong mode localization. That eigenloci veering phenomenon can occur in disordered structures under certain conditions has been explained qualitatively using geometric arguments in [6]; where it was also hinted that quantitative results might be obtainable by the use of singular perturbation analysis.

For mistuned linear chains, Pierre [5], and Pierre and Dowell [7] showed that the straightforward expansion in terms of mistuning parameters breaksdown in the case of weak coupling. This arises because the idealized system that is being perturbed has natural frequencies with multiplicity > 1. They then developed a so-called "modified" perturbation technique which provided a good approximation to the exact eigenfrequencies and showed good agreement with experimental results. In the case of strong coupling between identical subsystems, no such difficulty arises and regular perturbation expansions in terms of mistuning parameters are uniformly valid.

For systems with cyclic symmetry or spatial periodicity, however, mode localization can arise in the presence of perturbations which split the degenerate or coincident eigenvalues, irrespective of the strength of internal coupling [6]. Using differential topological ideas it was shown qualitatively in [6] that circularly configured systems which have cyclic symmetry exhibit complicated topological behavior even for strong coupling when small perturbations are imposed. Furthermore, the frequency response of a perturbed cyclic system depends significantly on the form of the perturbation. Such cyclic periodic systems are important to the analysis of vibrations of bladed disk assemblies.

In the present work we show that the singularity causing the breakdown of the straightforward expansion can be analyzed by the well developed singular perturbation techniques [8] and appropriate uniform asymptotic expansions for the eigenfrequencies and eigenvectors can be constructed which provide a correct qualitative and good quantitative approximation. Preliminary results on eigenvalue veering for the now standard example [5] of the coupled penduli shown in Figure 1 were recently reported in a short paper [9]. Here we present complete details of the singular perturbation analysis for the eigenvalue problem of the coupled penduli system. Using the uniform asymptotic expansions for eigenvectors, the occurance of mode localization is then related to sensitivity with respect to parameter variations. We then consider the simplest of examples of systems with cyclic symmetry consisting of three identical masses arranged in a ring, interconnected by identical springs and having individual torsional stiffnesses. The eigenvalue veering is here shown to exist even for the strong coupling case. Finally, based on the solutions for the strong coupling case, behavior for the weak coupling limit is explored.

The basic idea of analysis by the singular perturbation technique is the following: by applying the regular perturbation technique to the eigenvalue problem $A\phi = \lambda\phi$, of the system, we can obtain algebraic expressions for the eigenvalues and eigenfuctions as a power series in the small parameter or perturbation (say δ). The coefficients of the power series are dependent on a second parameter ε and these expansions are valid for sufficiently small δ, for all values of ε, so long as no singularities arise. Singularities occur for values of ε where the eigenfrequencies and eigenfunctions lose their smoothness and it is said that the expansion is not uniformly valid for all ε. Away from the singular values of the parameter (ε), the straightforward expansions are good approximations and are called the "outer expansions". The neighborhood of the singular parameter point is then stretched or rescaled in terms of a new parameter so as to remove the singularity. The expansion in terms of the new parameter is valid only in the neighborhood of the singular point and is called the "inner expansion". The inner and the outer solutions can be matched where their domains of validity overlap and then a composite expansion can be constructed which is valid uniformly throughout the function domain for all values of the parameter ε.

2. THE COUPLED PENDULI

2.1 Singular Perturbation Analysis

Consider the system consisting of two weakly coupled penduli as shown in Figure 1. The two important parameters are the dimensionless length Δl, and the dimensionless coupling between the two pendulums $\delta = R^2 = (k/m)/(g/l)$. The dimensionless parameter Δl represents the disorder or perturbation in the individual pendulums. It is important to point out that the dimensionless coupling between the two pendulums $\delta << 1$ for weak coupling irrespective of Δl. The resulting eigenvalue problem in symmetric form generated by the above system is given by

$$A\,\phi = \lambda\,\phi\,, \tag{1}$$

where $\delta = R^2 = \omega_k^2/\omega_g^2$, $\omega_k^2 = k/m$, $\omega_g^2 = g/l$,

$$A = \begin{bmatrix} 1+R^2 & -R^2 \\ -R^2 & R^2+(1+\Delta l)^{-1} \end{bmatrix} .$$

For small values of δ, it is natural to expand eigenvalues and eigenfuctions in the regular expansion as powers of δ regarding Δl as a parameter in the range of interest. Thus we write A, λ, and ϕ in powers of δ as

$$A = A_o + A_1\delta + A_2\delta^2 + O(\delta^3)\,, \tag{2}$$

$$\lambda = \lambda_o + \lambda_1\delta + \lambda_2\delta^2 + O(\delta^3)\,, \tag{3}$$

$$\phi = \phi_o + \phi_1\delta + \phi_2\delta^2 + O(\delta^3)\,. \tag{4}$$

Substituting (2), (3) and (4) into (1), equating coefficients of each power of δ to zero, and solving the resulting sequence of homogeneous and nonhomogeneous linear systems gives the following expansions for the eigenvalues and the corresponding eigenvectors

$$\lambda^1 = 1 + \delta + \left[(1 + \Delta l)/\Delta l\right] \delta^2 + O(\delta^3)\,, \tag{5}$$

$$\lambda^2 = 1/(1 + \Delta l) + \delta - \left[(1 + \Delta l)/\Delta l\right]\delta^2 + O(\delta^3)\,, \tag{6}$$

$$\phi^1 = \begin{bmatrix} C \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\left[(1 + \Delta l)/\Delta l\right]C \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \delta^2 + O(\delta^3)\,, \tag{7}$$

$$\phi^2 = \begin{bmatrix} 0 \\ C_1 \end{bmatrix} + \begin{bmatrix} -\left[(1 + \Delta l)/\Delta l\right]C_1 \\ 0 \end{bmatrix} \delta + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \delta^2 + O(\delta^3)\,. \tag{8}$$

In (7)-(8), C and C_1 are arbitrary constants. The expressions (5) - (8) are the regular (outer) expansions of the eigenvalue problem for small coupling δ which depend on the parameter Δl. These are valid for sufficiently small δ for all values of Δl. When $\Delta l \to 0$, (5) - (8) become unbounded and the continuity of eigenvalues and eigenvectors with respect to the perturbation Δl breaks down. Thus, in the neighborhood of $\Delta l = 0$ the expansions (5)-(8) become nonuniform and singular or non-analytic points have therefore been identified. These eigenvalues in (5) and (6) are plotted in Figure 2 for some small but fixed δ as a function of Δl. Each eigencurve has two branches, one valid for $\Delta l > 0$ and the other for $\Delta l < 0$. These branches are indicated by the superscripts '+' and '-' and correspond to $\Delta l > 0$ and $\Delta l < 0$, respectively. Note that since Δl and δ have been treated as two independent parameters there is no control over expressions (5) - (8) in the limiting process when $\Delta l \to 0$ and $\delta \to 0$. By stretching the neighborhood of the singular parameter value $\Delta l = 0$ and by taking into consideration the nature of the singularity we can find an exact relation between these two parameters. Then Δl and δ become dependent in the neighborhood of the singular parameter value, called the "inner region". The solutions of the problem in the inner region are called the inner expansions. Asymptotically matching the inner and the outer expansions, and combining them appropriately then gives the composite expansions which are valid throughout the interval of interest in Δl for sufficiently small δ. Before finding inner expansions for the eigenvalues and the eigenfunctions, we mass normalize eigenfunctions ϕ^1 and ϕ^2 to get a unique set of eigenfunctions

$$\phi_m^1 = \begin{bmatrix} \dfrac{-\Delta l}{\sqrt{\Delta l^2 + (1+\Delta l)^4 \delta^2}} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1+\Delta l}{\sqrt{\Delta l^2 + (1+\Delta l)^4 \delta^2}} \end{bmatrix} \delta + O(\delta^2) , \tag{9}$$

$$\phi_m^2 = \begin{bmatrix} 0 \\ \dfrac{\Delta l}{(1+\Delta l)\sqrt{\delta^2 + \Delta l^2}} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{\sqrt{\delta^2 + \Delta l^2}} \\ 0 \end{bmatrix} \delta + O(\delta^2) . \tag{10}$$

In the inner expansion, we assume that the physical parameters $R^2 = \delta$ and $\Delta l = \varepsilon$ are related (dependent) by a set of mathematical parameters $\xi_1, \xi_2, \xi_3, \ldots$ and μ through a "stretching" transformation of the form

$$\varepsilon = \varepsilon_o + \xi\mu^a + \sum_{j=0}^{\infty} \xi_j (\mu^a)^j , \tag{11}$$

where μ is a new small parameter that is defined by:

$$\delta(\mu) = (\mathrm{sgn}\delta)\mu^b . \tag{12}$$

For fixed μ, the quantity ξ serves as the internal variable. The positive constants a and b are to be determined by the nature of the singularities of the characteristic equation $F(\lambda, \varepsilon, \delta) = 0$ of (1) near $\varepsilon = \varepsilon_o$, where ε_o is the singular point of interest. Let the eigenfunctions ϕ and the eigenvalues λ be written, in the inner region, as the expansions

$$\phi(\varepsilon,\delta) = z(\varepsilon(\mu),\delta(\mu)) = z(\mu) = \sum_{j=0}^{\infty} z_j \mu^j \,, \tag{13}$$

$$\lambda(\varepsilon,\delta) = \lambda(\varepsilon(\mu),\delta(\mu)) = \Omega\,(\mu) = \sum_{j=0}^{\infty} \Omega_j \mu^j \,. \tag{14}$$

Note that for the pendulum problem $F(\lambda,\varepsilon,0) = 0$ has a singular point at $\varepsilon = \varepsilon_o = 0$. The expansions (13) and (14) are the "inner expansions" and the z_j's and Ω_j's are called the "inner coefficients". Substituting (11), (12), (13), and (14) into (1) simplifying the inner expansions with a = b = 1, and solving the sequence of eigenvalue problems obtained by equating each power of μ to zero, we obtain the following inner eigenvalues Ω^1, Ω^2 and inner eigenfunctions z^1 and z^2

$$\Omega^1 = 1 + \left[\frac{2-\xi+\sqrt{\xi^2+4}}{2}\right]\mu + \left[\frac{\xi^2}{1+\left[\frac{\xi+\sqrt{\xi^2+4}}{2}\right]^2}\right]\mu^2 + O(\mu^3)\,, \tag{15}$$

$$\Omega^2 = 1 + \left[\frac{2-\xi-\sqrt{\xi^2+4}}{2}\right]\mu + \left[\frac{\xi^2}{1+\left[\frac{-\xi+\sqrt{\xi^2+4}}{2}\right]^2}\right]\mu^2 + O(\mu^3)\,, \tag{16}$$

$$z^1 = a_o \begin{bmatrix} -k \\ 1 \end{bmatrix} + \begin{bmatrix} \dfrac{a_o \xi^2 k^2}{1+k^2} \\ 0 \end{bmatrix} \mu + O(\mu^2)\,, \tag{17}$$

$$z^2 = b_o \begin{bmatrix} m \\ 1 \end{bmatrix} + \begin{bmatrix} \dfrac{b_o \xi^2 m^2}{1+m^2} \\ 0 \end{bmatrix} \mu + O(\mu^2)\,, \tag{18}$$

where $\Delta l = \varepsilon = \xi\mu$, $\delta = R^2 = \mu$,

$$k = \frac{\xi+\sqrt{\xi^2+4}}{2}\,, \tag{19}$$

$$m = \frac{-\xi + \sqrt{\xi^2 + 4}}{2}, \tag{20}$$

$$a_o = \frac{1}{\sqrt{\left[-k + \frac{\xi^2 k^2 \mu}{1 + k^2}\right]^2 + (1 + \xi\mu)^2}}, \tag{21}$$

and

$$b_o = \frac{1}{\sqrt{\left[m + \frac{\xi^2 m^2 \mu}{1 + m^2}\right]^2 + (1 + \xi\mu)^2}}. \tag{22}$$

Here the eigenvectors z^1 and z^2 have been mass normalized. Keeping μ fixed and taking the limit $|\xi| \to \infty$, it is easy to see that Ω^1 matches asymptotically with λ_+^1 for $\xi \to \infty$, and with λ_-^2 for $\xi \to -\infty$. Similarly the inner solution Ω^2 matches asymptotically ($|\xi| \to \infty$) with λ_-^1 and λ_+^2. In fact, it can be easily shown that the outer eigenvectors ϕ_m^1 and ϕ_m^2 match with the inner eigenvectors z^1 and z^2 in exactly the way the eigenvalue branches match. The composite expansions for the eigenvalues and the eigenfunctions are now obtained by combining the inner and the outer expansions, and subtracting the common part of the two expansions. The resulting expressions for the eigenvalues and the eigenvectors are

$$\lambda_{comp}^1 = \left[\frac{1}{1+\Delta l} + \delta - \left(1 + \frac{1}{\Delta l}\right)\delta^2\right](1 - u(\Delta l)) + \left[1 + \delta + \left(1 + \frac{1}{\Delta l}\right)\delta^2\right]u(\Delta l)$$

$$+ \left\{1 + \left(\frac{2 - (\Delta l/\delta) + \sqrt{(\Delta l/\delta)^2 + 4}}{2}\right)\delta + \left[\frac{(\Delta l/\delta)^2}{1 + \left(\frac{(\Delta l/\delta) + \sqrt{(\Delta l/\delta)^2 + 4}}{2}\right)^2}\right]\delta^2\right\}$$

$$-(1-u(\Delta l))\,(1-\Delta l+\Delta l^2+\delta-(1+1/\Delta l)\delta^2)-(1+\delta+(1+1/\Delta l)\delta^2)u(\Delta l)+O(\delta^3)\,, \tag{23}$$

$$\lambda^2_{comp}=\left[1+\delta+\left[1+\frac{1}{\Delta l}\right]\delta^2\right](1-u(\Delta l))+\left[\frac{1}{1+\Delta l}+\delta-\left[1+\frac{1}{\Delta l}\right]\delta^2\right]u(\Delta l)$$

$$+\left\{1+\left[\frac{2-(\Delta l/\delta)-\sqrt{(\Delta l/\delta)^2+4}}{2}\right]\delta+\left[\frac{(\Delta l/\delta)^2}{1+\left[\frac{(\Delta l/\delta)+\sqrt{(\Delta l/\delta)^2+4}}{2}\right]^2}\right]\delta^2\right\}$$

$$-(1-u(\Delta l))(1+\delta+(1+\frac{1}{\Delta l})\delta^2)-u(\Delta l)[1-\Delta l+\Delta l^2+\delta-(1+\frac{1}{\Delta l})\delta^2]+O(\delta^3)\,, \tag{24}$$

$$\phi^1_{comp}=z^1+u(\Delta l)\phi^1_m+(1-u(\Delta l))\phi^2_m-u(\Delta l)\begin{bmatrix}-1+\delta/2\Delta l\\ \delta/\Delta l-\dfrac{\delta^3}{2\Delta l^3}\end{bmatrix}+\begin{bmatrix}\dfrac{2\delta}{\Delta l}\\ 1-\dfrac{5\delta^2}{2\Delta l}\end{bmatrix}$$

$$-\left[1-u(\Delta l)\right]\left\{\begin{bmatrix}\dfrac{\delta}{\Delta l\left[1+\dfrac{\delta^2}{2\Delta l^2}\right]}\\ \dfrac{1}{1+\dfrac{\delta^2}{2\Delta l^2}}\end{bmatrix}+\begin{bmatrix}0\\ \dfrac{-\Delta l}{\delta(1+\dfrac{\delta^2}{2\Delta l^2})}\end{bmatrix}\delta\right\}+O(\delta^2)\,,$$

which simplifies to

$$\phi^1_{comp}=a_o\left\{\begin{bmatrix}-k\\ 1\end{bmatrix}+\begin{bmatrix}\dfrac{k^2\Delta l^2/\delta^2}{1+k^2}\\ 0\end{bmatrix}\delta\right\}+O(\delta^2)\,, \tag{25}$$

and

$$\phi^2_{comp} = b_o \left\{ \begin{bmatrix} m \\ 1 \end{bmatrix} + \begin{bmatrix} \dfrac{m^2 \Delta l^2/\delta^2}{1+m^2} \\ 0 \end{bmatrix} \delta \right\} + O(\delta^2) , \tag{26}$$

where k, m, a_o, and b_o are already defined in equations (19), (20), (21) and (22) respectively and,

$$u(\Delta l) = \begin{cases} 1 , & \Delta l \geq 0 \\ 0 , & \Delta l < 0 \end{cases} .$$

The composite solutions (23) - (26) of the eigenvalue problem (1) are the asymptotic approximations for small coupling δ, and are uniformly valid for all mistunings Δl.

The eigenfrequencies λ^1_{comp} and λ^2_{comp}, obtained by the singular perturbation analysis, are plotted in Figure 3 as a function of the parameter Δl. Since the coupled penduli system is simple, exact expressions for the eigenfrequencies are easily obtained and they are also plotted in the figure. The exact solutions for the eigenvalue problem are given in the Appendix. Clearly, there is an excellent agreement in the exact frequencies and their asymptotic approximations. The asymptotic solutions also clearly display the veering phenomenon.

We now study the behavior of eigenvectors for the case of weak coupling when $\Delta l \sim 0$.

2.2 Eigenvector Rotations and the Sensitivity Function

In our earlier work [6] with mistuned cyclic systems, it was suggested that localization of modes can be investigated by considering the sensitivity function, and the rotations of eigenvectors under variations of parameters. The sensitivity function of eigenvectors or eigenvector sensitivity, in short, is defined as

$$||\, S_u \,|| = \sqrt{\mathrm{tr}(S_u^* S_u)} \quad \text{(Frobenius norm)},$$

where $S_u = u_o^{-1}\Delta u$, S_u^* denotes the complex conjugate of transpose of S_u, u_o is the modal matrix of eigenvectors for zero coupling ($\delta = 0$), 'tr' denotes the trace of the matrix, and $\Delta u = u - u_o$ where u is the modal matrix for nonzero coupling ($\delta \neq 0$). The eigenvector sensitivity evaluated for the pendulum problem turns out to be

$$||\, S_u \,|| = \frac{1}{\sqrt{2}} \sqrt{(-P_{11} + P_{21})^2 + (P_{11} + P_{21})^2 + (-P_{12} + P_{22})^2 + (P_{12} + P_{22})^2}\ , \tag{27}$$

where

$$P_{11} = -a_o k + \frac{1}{\sqrt{2}} + \frac{a_o k^2 (\Delta l)^2}{\delta(1 + k^2)}\ ,$$

$$P_{12} = b_o m - \frac{1}{\sqrt{2}} + \frac{b_o m (\Delta l)^2}{\delta(1 + m^2)}\ ,$$

$$P_{21} = a_o - \frac{1}{\sqrt{2}}\ ,$$

$$P_{22} = b_o - \frac{1}{\sqrt{2}}\ .$$

We can also define the angles between the eigenvectors ϕ_o^1 and ϕ_o^2 for the unperturbed ($\delta = 0$) system, and the eigenvectors ϕ_{comp}^1 and ϕ_{comp}^2 for the perturbed system. These angles are given by

$$\cos\theta_1 = \frac{\langle \phi_o^1 \,,\, \phi_{comp}^1 \rangle}{||\,\phi_o^1\,||\;||\,\phi_{comp}^1\,||} = \frac{1 - P_1}{\sqrt{2}\sqrt{1 + P_1^2}}\ , \tag{28}$$

$$\cos\theta_2 = \frac{<\phi_o^2 \, , \, \phi_{comp}^2>}{\|\phi_o^2\| \, \|\phi_{comp}^2\|} = \frac{1 - P_2}{\sqrt{2}\sqrt{1 + P_2^2}} \, ,$$

where

$$P_1 = k + \frac{k^2(\Delta l)^2}{\delta(1+k^2)} \, ,$$

$$P_2 = m + \frac{m^2(\Delta l)^2}{\delta(1+m^2)} \, .$$

Plots of the sensitivity function of eigenvectors and the cosine of the angle between nominal ($\delta = 0$) and perturbed eigenvectors, as a function of the mistuning Δl, are given in Figures 4 and 5, for both the exact solutions and the asymptotic approximations obtained above by the singular perturbation technique. These solutions are in excellent agreement. Figures 4 and 5 clearly show and confirm the expectation that the eigenvectors for the weakly coupled system undergo rapid changes in the vicinity of the singular point. Furthermore, either of the two criterion can be effectively used as a quantitative measure and indicator of the mode localization phenomenon.

3 CYCLIC SYSTEMS

Consider three identical particles, each of mass m arranged in a ring and interconnected by identical springs of stiffness k_c. Assume that all the masses are hinged to the ground by torsional springs of stiffness k_t and that the radius of the ring is r, as shown in Figure 6. As a perturbed system we consider the case when two of the torsional springs are perturbed by ε_1 and ε_2. The eigenvalue problem corresponding to this system is given by

$$\begin{bmatrix} a+\varepsilon_1 & -\omega_c^2 & -\omega_c^2 \\ -\omega_c^2 & a+\varepsilon_2 & -\omega_c^2 \\ -\omega_c^2 & -\omega_c^2 & a \end{bmatrix} \phi = \lambda\phi , \tag{29}$$

where $a = \dfrac{2k_c}{m} + \dfrac{k_t}{mr^2}$ and $\omega_c^2 = \dfrac{k_c}{m}$.

As was the case for the example of linear chains, the interest here is in the development of asymptotic expansions for the eigenvalues and eigenvectors in terms of the perturbation parameters ε_1 and ε_2. As we shall see the unperturbed cyclic system ($\varepsilon_1 = \varepsilon_2 = 0$), has a double eigenvalue and an isolated eigenvalue, and thus, introduction of perturbations is expected to split the degenerate eigenvalue pair. Expecting that the eigenvector behavior will be governed by the eigenvalue behavior, similar to the case of the coupled penduli, we restrict the developments to expansions for only the eigenvalues of the perturbed three particle system.

The eigenvalue problem (29) results in the characteristic equation

$$F(\lambda,\varepsilon_1,\varepsilon_2) = \lambda^3 - (3a+\varepsilon_1+\varepsilon_2)\lambda^2 + (3a^2+2a\varepsilon_2+2a\varepsilon_2+\varepsilon_1\varepsilon_2-3\omega_c^4)\lambda$$

$$+ \left[2\omega_c^6 + \omega_c^4(3a+\varepsilon_1+\varepsilon_2) - (a^2+a\varepsilon_1)(a+\varepsilon_2)\right] = 0 . \tag{30}$$

First consider the unperturbed cyclic system the eigenvalues or roots of (30) are given by

$$\bar{\lambda}^1 = \bar{\lambda}^2 = a + \omega_c^2 \ , \ \bar{\lambda}^3 = a - 2\omega_c^2 .$$

The corresponding roots of (31) are

$$\bar{\chi}^1 = \bar{\chi}^2 = \omega_c^2 \ , \ \bar{\chi}^3 = -2\omega_c^2 .$$

Thus, there is a coincident pair of eigenvalues and one isolated eigenvalue. So long as $\omega_c^2 \sim O(1)$, the two distinct eigenvalue are well separated.

To study the perturbed problem we first introduce the coordinate transformation $\chi = \lambda - a$, so that equation (30) results in

$$\chi^3 - (\varepsilon_1+\varepsilon_2)\chi^2 + (\varepsilon_1\varepsilon_2 - 3\omega_c^4)\chi + \omega_c^4(2\omega_c^2 + \varepsilon+\varepsilon_2) = 0 \tag{31}$$

We can express the solutions to (31) as regular functions of the parameters ε_1, ε_2, and ω_c^2 by writing $\chi(\varepsilon_1,\varepsilon,) = \sum_{j=0}^{\infty}\chi_j(\varepsilon_1)\varepsilon_2^j$ as a power series in ε_2. Substituting the resulting expression in (31) and proceeding in the usual manner, the expansions for the three roots of (31) turn out to be

$$\chi^1 = \omega_c^2 + \frac{\varepsilon_2}{2} + \frac{1}{8}\left[\frac{\omega_c^2-\varepsilon_1}{\omega_c^2\varepsilon_1}\right]\varepsilon_2^2 + O(\varepsilon_2^3)\,, \tag{32}$$

$$\chi^2 = \chi_{02} + \chi_{12}\varepsilon_2 + \chi_{22}\varepsilon_2^2 + O(\varepsilon_2^3)\,, \tag{33}$$

$$\chi^3 = \chi_{03} + \chi_{13}\varepsilon_2 + \chi_{23}\varepsilon_2^2 + O(\varepsilon_2^3)\,, \tag{34}$$

where

$$\chi_{02} = \frac{\varepsilon_1 - \omega_c^2 + P_1}{2}\ , \ \chi_{12} = \frac{\chi_{02}^2-\chi_{02}\varepsilon_1 - \omega_c^4}{3\chi_{02}^2-2\chi_{02}\varepsilon_1 - 3\omega_c^4}\,,$$

$$\chi_{22} = \frac{\chi_{12}^2(\varepsilon_1-3\chi_{02}) + \chi_{12}(2\chi_{02}-\varepsilon_1)}{3\chi_{02}^2-2\chi_{02}\varepsilon_1 - 3\omega_c^4}\,,$$

$$\chi_{03} = \frac{\varepsilon_1-\omega_c^2 - P_1}{2}\ , \ \chi_{13} = \frac{\chi_{03}^2-\chi_{03}\varepsilon_1 - \omega_c^4}{3\chi_{03}^2-2\chi_{03}\varepsilon_1 - 3\omega_c^4}\,,$$

$$\chi_{23} = \frac{\chi_{13}^2(\varepsilon_1-3\chi_{02}) + \chi_{13}(2\chi_{03}-\varepsilon_1)}{3\chi_{03}^2-2\chi_{03}\varepsilon_1 - 3\omega_c^4}\,,$$

and

$$P_1 = \sqrt{9\omega_c^4 + 2\omega_c^2\varepsilon_1 + \varepsilon_1^2}\ .$$

The expressions (32) - (34) are the straightforward expansions of the eigenvalue problem for small ε_2. When $\varepsilon_1 \to 0$, χ^1 and χ^2 become unbounded and the continuity of the eigenvalues with respect to the parameter or perturbation ε_1 breaks down. The third eigenvalue χ^3 always remain bounded and continuity is preserved for all values of ε_1 so long as the interconnecting or coupling spring constant k_c is 0(1). These eigenvalues (32)-(34) are shown in Figure 7. Clearly $\varepsilon_1 = 0$, $\varepsilon_2 = 0$ is a singular point of the expansions and (32)-(34) are the outer expansions valid for small ε_2 away from $\varepsilon_1 = 0$.

Inner expansions, which are valid in the neighborhood of singular point or the parameter values where the outer expansions breakdown are now obtained. The expansion process for the cyclic system is very similar to the one presented in section 2 for the linear chain system. Thus, the physical parameter perturbations ε_1 and ε_2 are related to a set of parameters ξ_1, ξ_2, ξ_3, and μ via a "stretching" transformation of the form

$$\varepsilon = \xi\mu^a + \sum_{j=0}^{\infty} \xi_j (\mu^a)^j\ , \tag{35}$$

where μ is a new small parameters that is defined by

$$\delta(\mu) = (\text{sgn}\ \delta)\mu^b\ . \tag{36}$$

The positive constants a and b are to be determined by the nature of the singularities of $F(\lambda, \varepsilon_1, \varepsilon_2) = 0$ near $\varepsilon_1 = 0$, the singular point of interest. Let the dependent variable or the eigenvalue $\Omega(\mu) = \Omega(\varepsilon_1(\mu), \varepsilon_2(\mu))$ be written as the expansion

$$\Omega = \sum_{j=0}^{\infty} \Omega_j \mu^j\ . \tag{37}$$

Note that for the cyclic system $F(\lambda,\varepsilon_1,\varepsilon_2=0)$ has a singular point at $\varepsilon_1=\varepsilon_2=0$. The expansions (37) are called the "inner expressions" and the Ω_js are called the "inner coefficients". Substituting (35), (36) and (37) into (30), simplifying the inner expansions with a=b=1, and performing the perturbation analysis, the following roots of (30) are obtained

$$\Omega^1=\omega_c^2+\Omega_{11}\,\mu+\left[\frac{(\xi+1)\Omega_{11}^2-\Omega_{11}^3-\xi\,\Omega_{11}}{2\omega_c^2\sqrt{\xi^2-\xi+1}}\right]\mu^2+O(\mu^3)\,, \tag{38}$$

$$\Omega^2=\omega_c^2+\Omega_{12}\,\mu+\left[\frac{\Omega_{12}^3+\xi\Omega_{12}-(\xi+1)\Omega_{12}^2}{2\omega_c^2\sqrt{\xi^2-\xi+1}}\right]\mu^2+O(\mu^3)\,, \tag{39}$$

$$\Omega^3=-2\omega_c^2+\left[\frac{\xi+1}{3}\right]\mu+\left[\frac{-2}{27\omega_c^2}\right](\xi^2-\xi+1)\mu^2+O(\mu^3)\,, \tag{40}$$

where

$$\varepsilon_1=\xi\mu\ ,\ \ \varepsilon_2=\mu\,,\ \ \Omega_{11}=\frac{(\xi+1)+\sqrt{\xi^2-\xi+1}}{3}\ \text{ and }\ \Omega_{12}=\frac{(\xi+1)-\sqrt{\xi^2-\xi+1}}{3}\,.$$

Keeping μ fixed and taking the limit $|\xi|\to\infty$, it is easy to see that Ω^1 matches asymptotically with χ_+^2 for $\xi\to\infty$ and with χ_-^1 for $\xi\to-\infty$. Similarly the inner solution Ω^2 matches asymptotically ($|\xi|\to\infty$) with χ_+^1 and χ_-^2. As expected, the third root Ω^3 automatically matches with χ^3 as no singular behavior is displayed in this case. Now combining the inner and the outer expansions appropriately, we get the composite expansions

$$\chi^1_{comp}=\chi^2(u(\varepsilon_1))+\chi^1(1-u(\varepsilon_1))+\Omega^1-u(\varepsilon_1)\ (\text{common parts of }\chi^2\ ,\ \Omega^1)$$

$$-(1-u(\varepsilon_1))\ (\text{common parts of }\chi^1\ ,\ \Omega^1)\,,$$

$$\chi^2_{comp} = \chi^1(u(\varepsilon_1)) + \chi^2(1-u(\varepsilon_1)) + \Omega^2 - u(\varepsilon_1)\text{ (common parts of } \chi^1\text{ , } \Omega^2)$$

$$-(1-u(\varepsilon_1))(\text{common parts of } \chi^2, \Omega^2)\ ,$$

$$\chi^3_{comp} = \chi^3\ ,$$

$$\text{where} \quad u(\varepsilon_1) = \begin{cases} 1\ , & \varepsilon_1 \geq 0 \\ 0\ , & \varepsilon_1 < 0 \end{cases}.$$

Transforming these expansions back into the original coordinates gives the following composite expansions for eigenvalues

$$\lambda^1_{comp} = a+\omega_c^2 + \Omega_{11}\ \varepsilon_2$$

$$+ \left[\frac{(1+\varepsilon_1/\varepsilon_2)\Omega_{11}^2 - \Omega_{11}^3 - (\varepsilon_1/\varepsilon_2)\Omega_{11}}{2\omega_c^2\sqrt{(\varepsilon_1/\varepsilon_2)^2 - (\varepsilon_1/\varepsilon_2) + 1}} \right] \varepsilon_2^2 + O(\varepsilon_2^3)\ , \tag{41}$$

$$\lambda^2_{comp} = a+\omega_c^2 + \Omega_{12}\ \varepsilon_2 +$$

$$\left[\frac{\Omega_{12}^3 + \varepsilon_1/\varepsilon_2\Omega_{12} - (\varepsilon_1/\varepsilon_2 + 1)\Omega_{12}^2}{2\omega_c^2\sqrt{(\varepsilon_1/\varepsilon_2)^2 - (\varepsilon_1/\varepsilon_2) + 1}} \right] \varepsilon_2^2 + O(\varepsilon_2^3)\ , \tag{42}$$

$$\lambda^3_{comp} = \chi_{03} + (\chi_{13})\varepsilon_2 + \chi_{23}(\varepsilon_2^2) + O(\varepsilon_2^3)\ , \tag{43}$$

where

$$\Omega_{11} = \frac{(\varepsilon_1/\varepsilon_2 + 1) + \sqrt{(\varepsilon_1/\varepsilon_2)^2 - (\varepsilon_1/\varepsilon_2) + 1}}{3}\ ,$$

$$\Omega_{12} = \frac{(\varepsilon_1/\varepsilon_2 + 1) - \sqrt{(\varepsilon_1/\varepsilon_2)^2 - (\varepsilon_1/\varepsilon_2) + 1}}{3}\ ,$$

$$\chi_{13} = \frac{\chi_{03}^2 - \chi_{03}\varepsilon_1 - \omega_c^4}{3\chi_{03}^2 - 2\chi_{03}\varepsilon_1 - 3\omega_c^4},$$

$$\chi_{03} = \frac{\varepsilon_1 - \omega_c^4 - P_1}{2},$$

$$P_1 = \sqrt{9\omega_c^4 + 2\omega_c^2\varepsilon_1 + \varepsilon_1^2}.$$

The plots of eigenfrequencies λ^1_{comp}, λ^2_{comp} and λ^3_{comp} as a function of ε_1 are given in Figure 8a and 8b. The third eigenvalue λ^3_{comp} always behaves as a regular function as is clear from Figure 8a. These results show that for the cyclic system with strong coupling the effect of perturbation is identical to that for the pendulum problem with weak coupling. That is, the curve veering of the eigenvalues λ^1_{comp} and λ^2_{comp} is almost the same as the curve veering of the weakly coupled pendulum. Consequently, it is expected that the mode localization of the eigenvectors within the strongly coupled cyclic system be similar to that in the weakly coupled pendulum problem. Therefore, the results should be obtained for the singular behavior of the eigenvectors of the weakly coupled pendulum problem in section 2 should be valid quatitatively for the cyclic system.

The coupling constant k_c plays a very important role in the eigenvalue veering behavior. For the cyclic system, $k_c = 0$ leads to three coincident eigenvalues as opposed to the case of strong coupling when only a double eigenvalue appears. The composite expansions obtained earlier were determined under the assumption that $k_c \sim O(1)$. We now use these expansions and explore the behavior of eigenvalues in the limiting ease of $k_c \to 0$. Note that the coupling constant ω_c^2 appear in the denominator of the asymptotic expansions (41)-(43). As ω_c^2 is reduced the isolated eigenvalue λ^3_{comp} moves closer to λ^1_{comp} and λ^2_{comp} and the variation of λ's with ε_2

becomes very rapid, as is evident from Figures 9 and 10. Figure 11 shows the three composite eigenvalue expansions for small ε_1 and ε_2 as a function of k_c (or ω_c^2) and the expansions clearly breakdown in the vicinity of $k_c = 0$. Thus, the composite expansions (41)-(43) are behaving as outer expansions in the weak coupling limit. It should be possible to now construct uniformly valid expansions for eigenvalues as a function of the coupling constant k_c by one more use of the singular perturbation technique whereby the neighborhood of $k_c = 0$ is stretched and an inner expansion is developed. The localization and veering behavior of thus obtained composite eigenvalue expansions, which will be valid for small enough ε_2 for all ε_1 and k_c, is expected to be much more interesting and is being presently studied.

4. SUMMARY AND CONCLUSIONS

Singular perturbation technique has been applied to two parameter eigenvalue problems to obtain uniformly valid algebraic expansions for the eigenvalues and the eigenvectors for two example systems. Utilizing these expansions, eigenloci veering and the mode localization phenomenon have been studied. A sensitivity function and the rotation of eigenvectors have been introduced as criteria to visualize mode localization phenomenon in the vicinity of singular points. One example, that of the two weakly coupled penduli, represents systems of the linear chain type with only one weak coupling spring. The example of three mass particles belongs to strongly coupled systems with cyclic symmetry. For the coupled penduli system, the eigenvalue and eigenvector expansions are found to be in excellend agreement with the exact results.

It is shown that eigenvalue curve veering occurs both in the weakly coupled penduli and the strongly coupled cyclic system. The effects of mistuning perturbations which split a pair of coincident eigenvalues is identical in both the cases. The eigenvector sensitivity function and the angle of rotation of eigenvectors are shown to be two equally good candidates for visualizing mode localization phenomenon near singular points. The composite expansions for the perturbed cyclic system, which are uniformly valid in the case of strong coupling, are shown to breakdown in the limiting case when the coupling stiffness goes to zero. This clearly is related to the fact that all the eigenvalues for the cyclic system in the weak coupling limit are identical.

ACKNOWLEDGEMENT

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APPENDIX

It can be easily shown that the two free vibration natural frequencies ω_i^2 and the corresponding mass normalized eigenvectors x^i for the coupled pendula problem, obtained from the exact solution of the eigenvalue problem, are given by

$$2\omega_{1,2}^2 = 2R^2 + 1 + \frac{1}{1+\Delta l} \mp \left[4R^4 + 1 - \frac{2}{1+\Delta l} + \frac{1}{(1+\Delta l)^2}\right]^{1/2},$$

$$x^1 = \alpha \begin{bmatrix} \chi_{11}^1 \\ 1 \end{bmatrix},$$

$$x^2 = \beta \begin{bmatrix} \chi_{11}^2 \\ 1 \end{bmatrix},$$

where $\alpha = \dfrac{1}{\sqrt{(\chi_{11}^1)^2 + (1 + \Delta l)^2}}$,

$$\beta = \frac{1}{\sqrt{(\chi_{11}^{2})^2 + (1+\Delta l)^2}} ,$$

$$\chi_{11}^{1} = \frac{1}{2\delta}\left[1 - \frac{1}{(1+\Delta l)} - \left\{4R^4 + \left[\frac{\Delta l}{1+\Delta l}\right]^2\right\}^{1/2}\right],$$

$$\chi_{11}^{2} = \frac{1}{2\delta}\left[1 - \frac{1}{(1+\Delta l)} + \left\{4R^4 + \left[\frac{\Delta l}{1+\Delta l}\right]^2\right\}^{1/2}\right].$$

The sensitivity of eigenvectors is then given by

$$||S_v^T|| = \frac{1}{\sqrt{2}}\sqrt{(-q_{11}+q_{21})^2 + (q_{11}+q_{21})^2 + (-q_{12}+q_{22})^2 + (q_{12}+q_{22})^2} ,$$

where

$$q_{11} = \alpha\,\chi_{11}^{1} + \frac{1}{\sqrt{2}} ,$$

$$q_{12} = \beta\,\chi_{11}^{2} - \frac{1}{\sqrt{2}} ,$$

$$q_{21} = \alpha - \frac{1}{\sqrt{2}} ,$$

$$q_{22} = \beta - \frac{1}{\sqrt{2}} ,$$

The angles between the nominal and the perturbed eigenvectors are then

$$\cos\theta_1^T = \frac{1-\chi_{11}^1}{\sqrt{2}\sqrt{1+(\chi_{11}^1)^2}}\,,$$

$$\cos\theta_2^T = \frac{1-\chi_{11}^2}{\sqrt{2}\sqrt{1+(\chi_{11}^2)^2}}\,.$$

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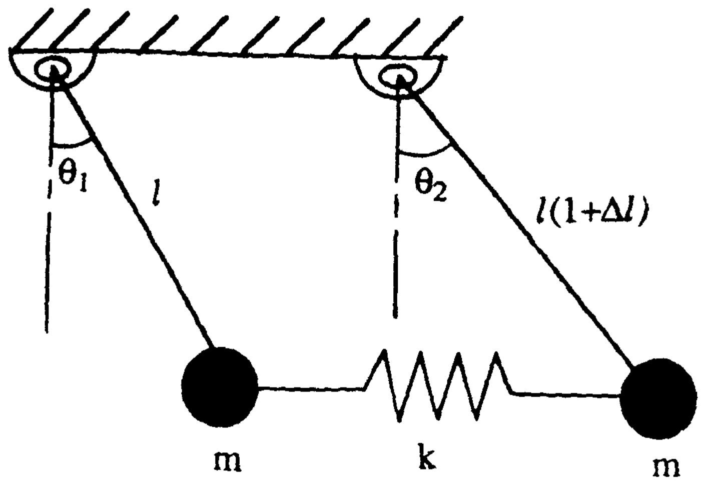

Figure 1. Two coupled oscillators.

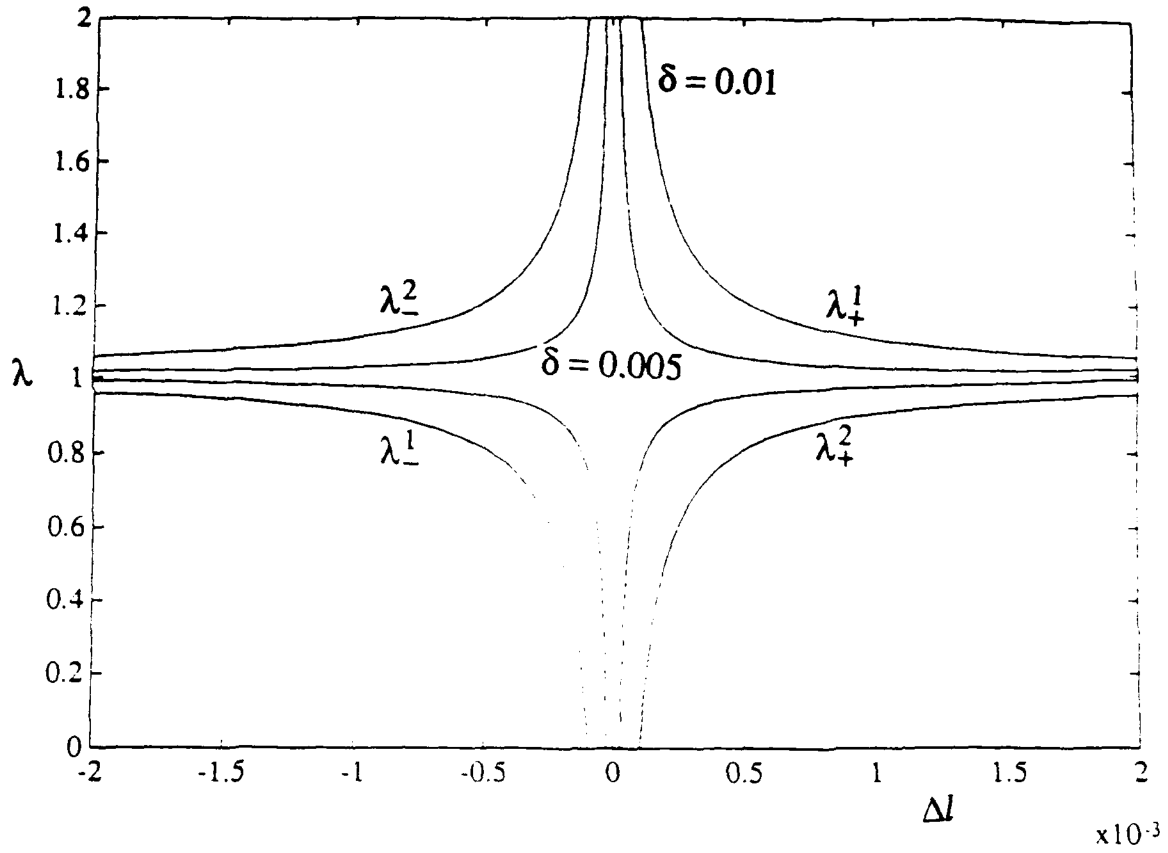

Figure 2. Outer expansions for eigenvalues indicating the region of singular behavior; $\delta = 0.01$, $\delta = 0.005$.

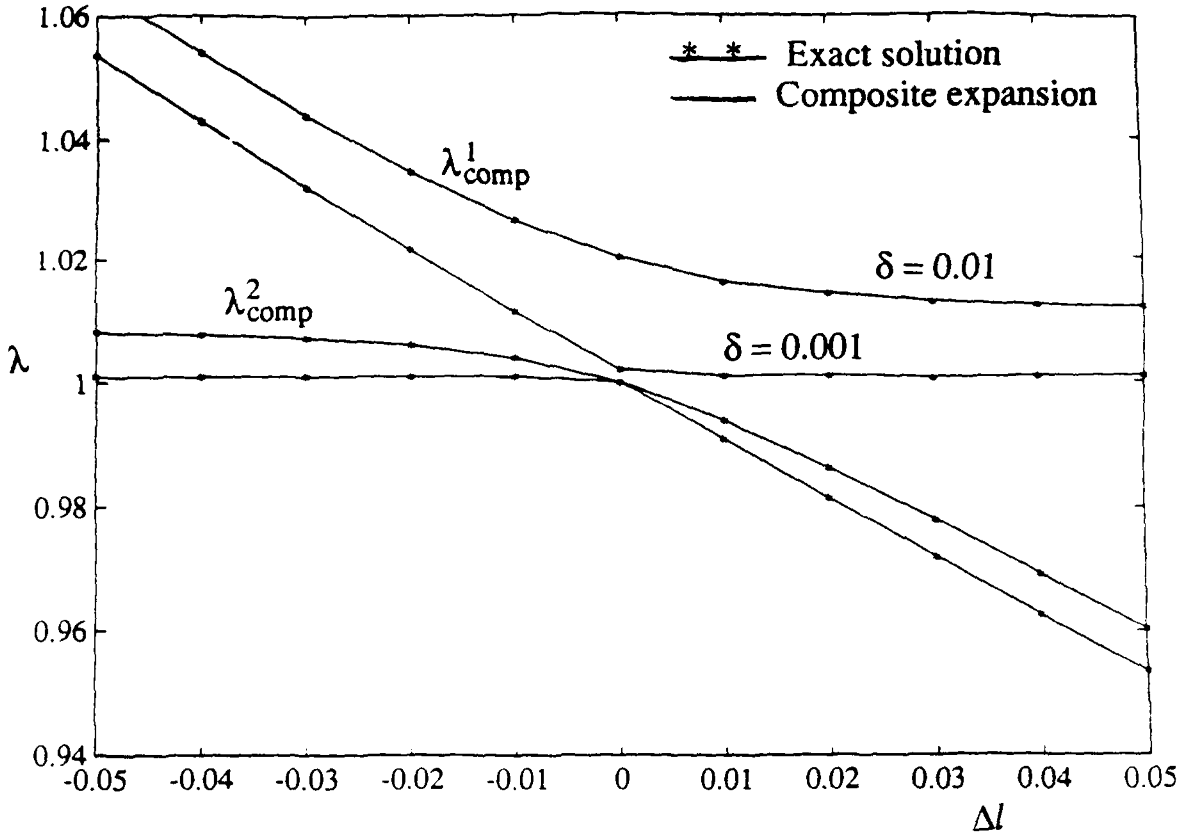

Figure 3. Comparison of the exact eigenvalues with those obtained from the composite expansions; $\delta = 0.01$, $\delta = 0.001$.

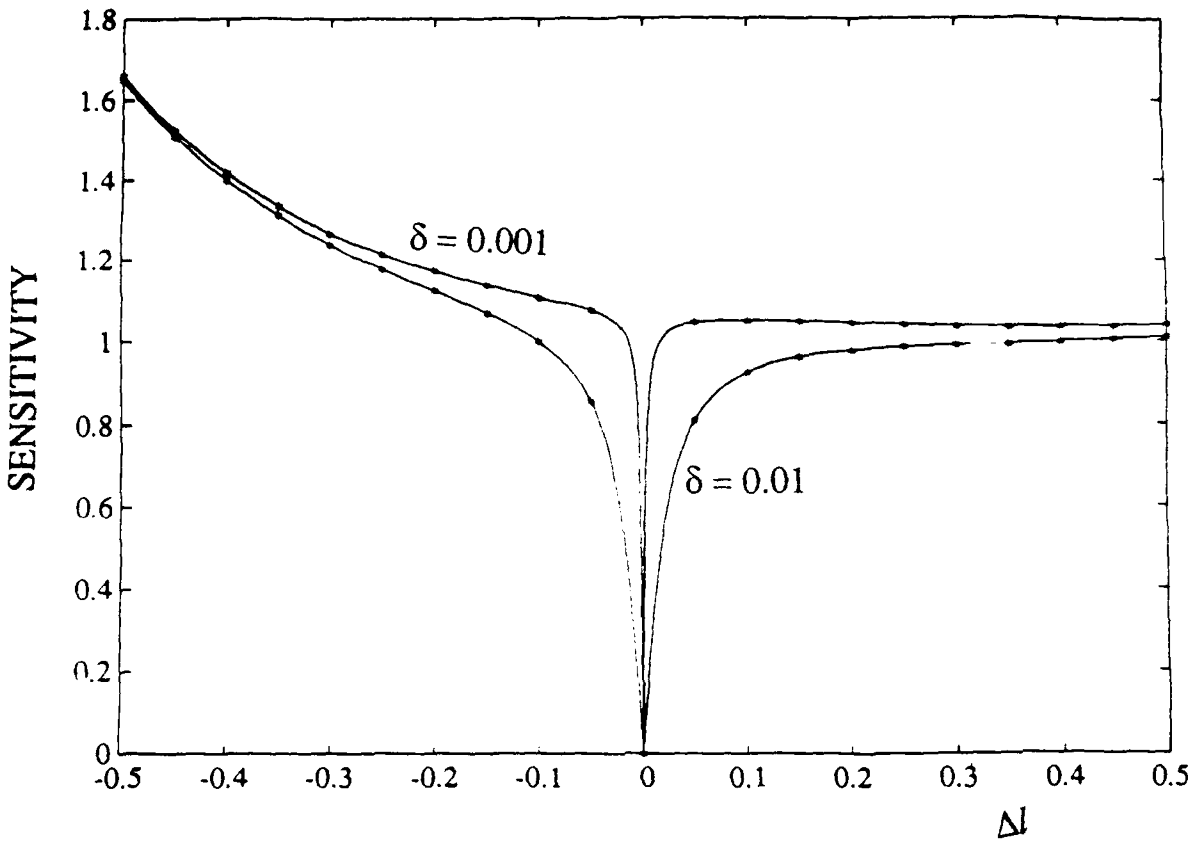

Figure 4. Comparison of the exact eigenvector sensitivity with that evaluated using the composite expansions; $\delta = 0.01$, $\delta = 0.001$.

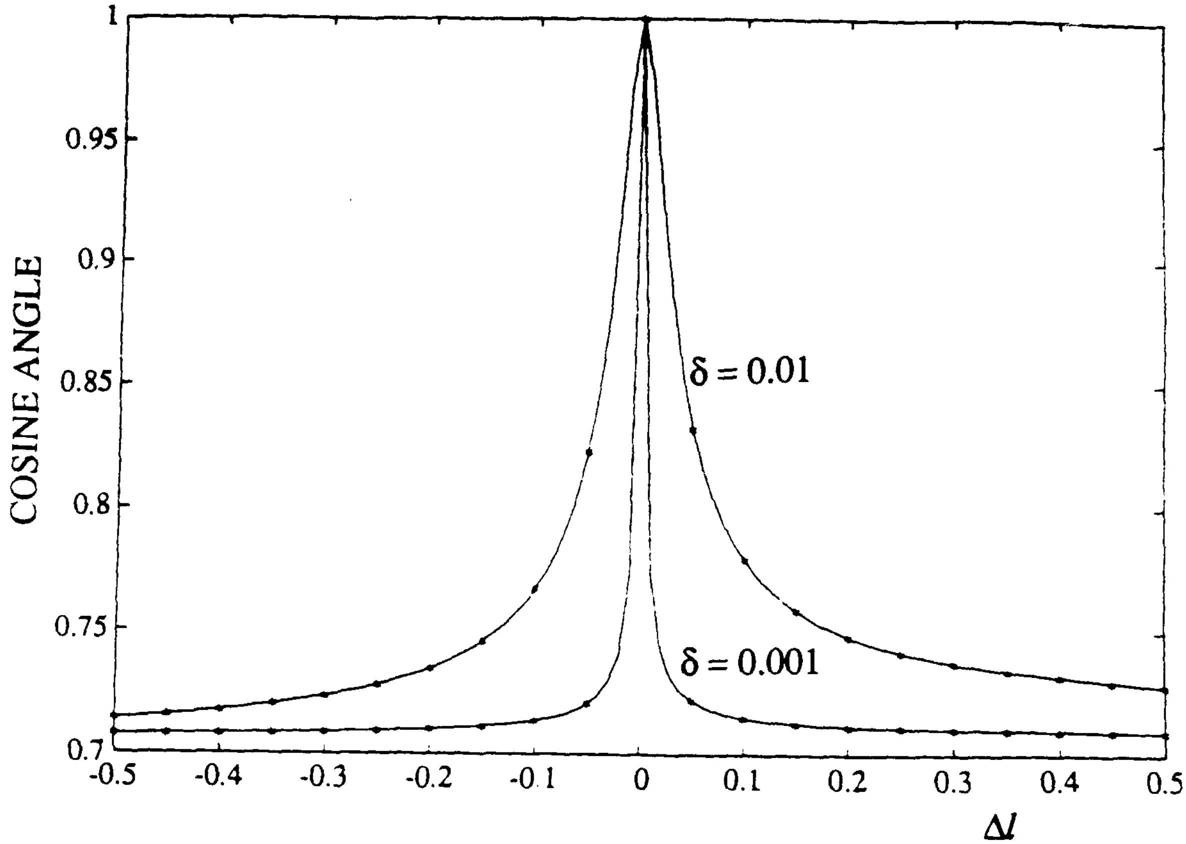

Figure 5. Comparison of the exact eigenvector rotations with those obtained from the composite expansions; $\delta = 0.01$, $\delta = 0.001$.

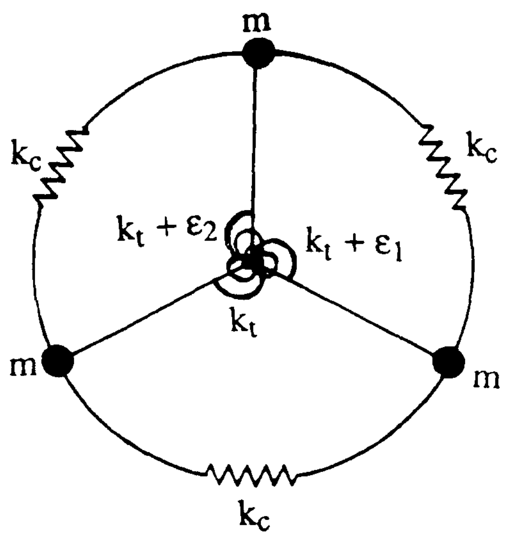

Figure 6. Model of a three bladed disk assembly.

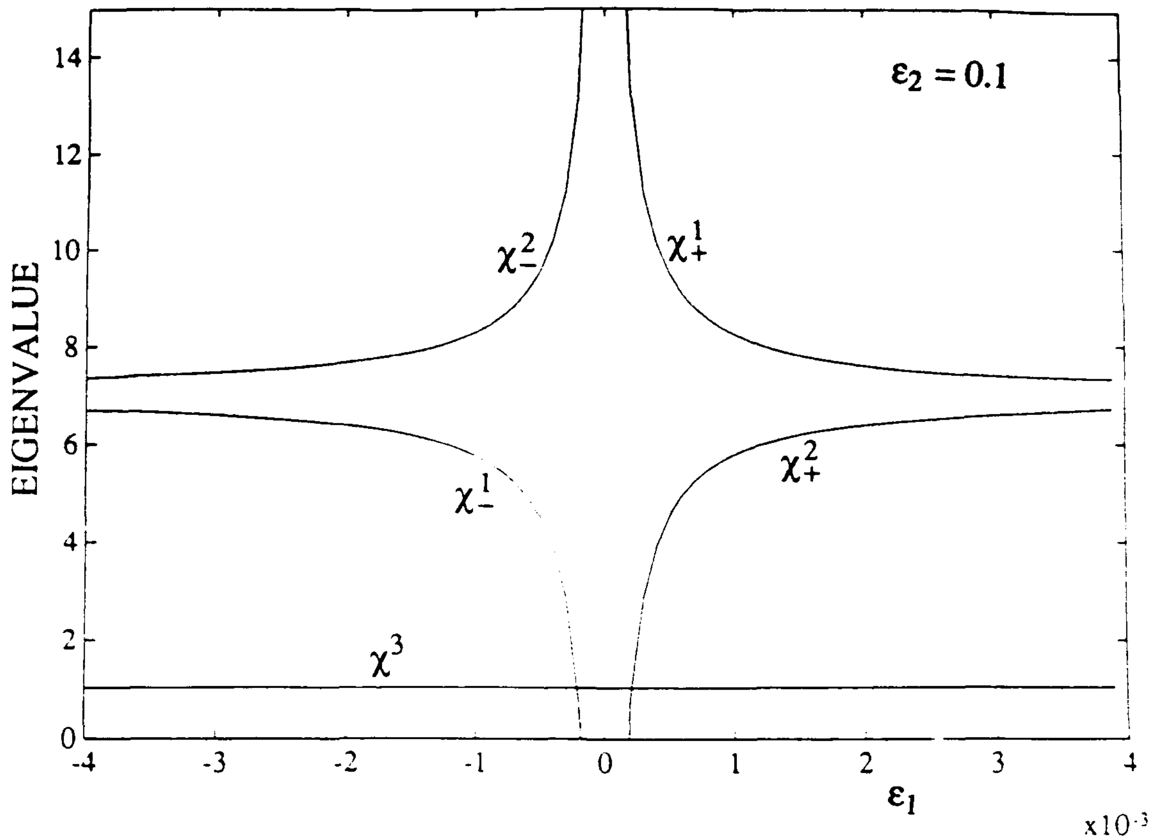

Figure 7. Outer expansions for the eigenvalues of the perturbed cyclic system indicating the region of singular behavior; $k_c = 2$, $k_t = 1$, $\varepsilon_2 = 0.1$, $r = 1$, $m = 1$.

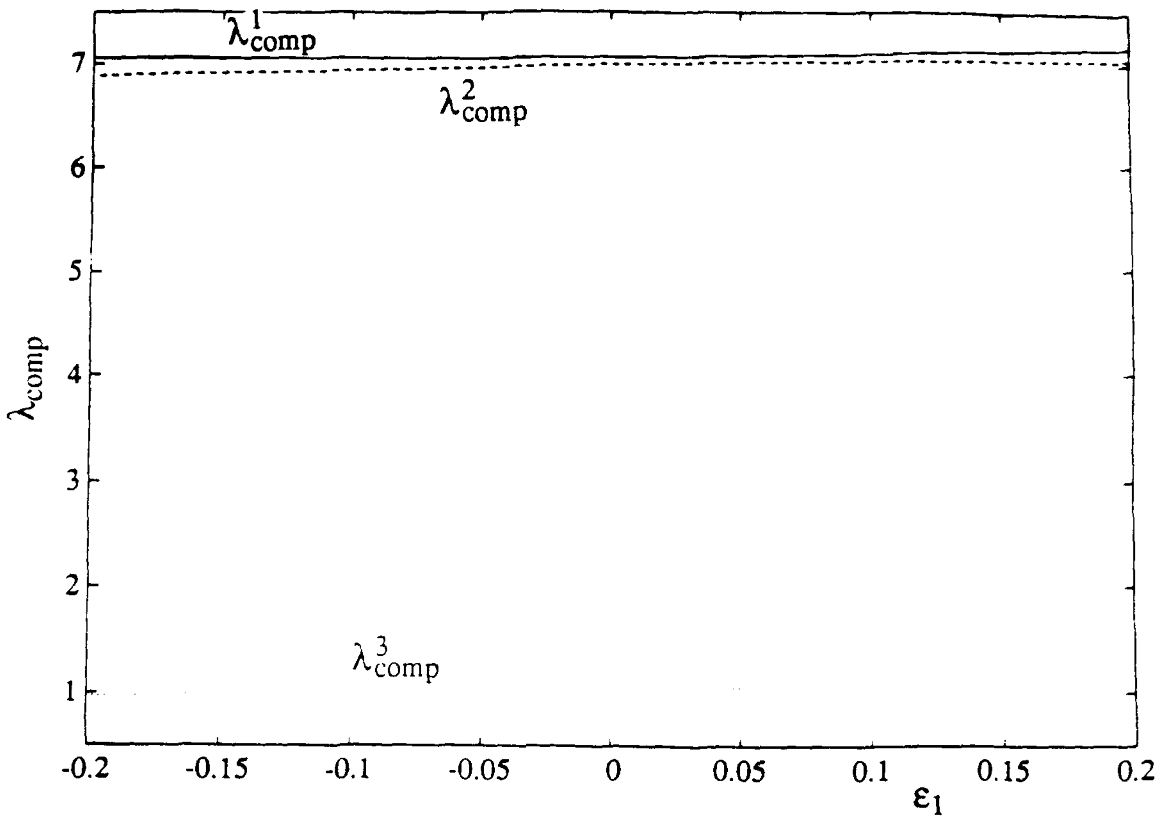

Figure 8a. Eigenvalues from composite expansions for the perturbed cyclic system; $k_c = 2$, $k_t = 1$, $\varepsilon_2 = 0.1$, $m = 1$, $r = 1$.

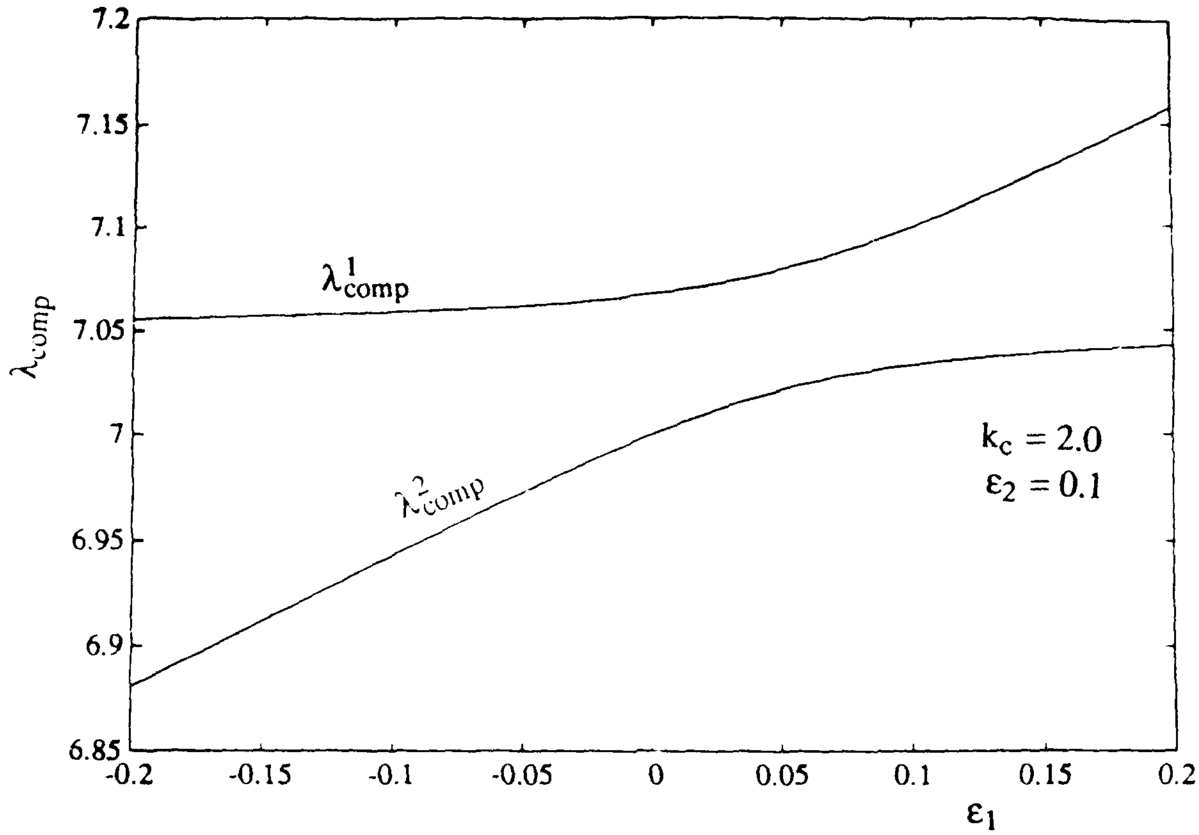

Figure 8b. Behavior of the two eigenvalues λ^1_{comp}, λ^2_{comp} showing curve veering for the strong coupling case; $k_c = 2$, $k_t = 1$, $\varepsilon_2 = 0.1$, $m = 1$, $r = 1$.

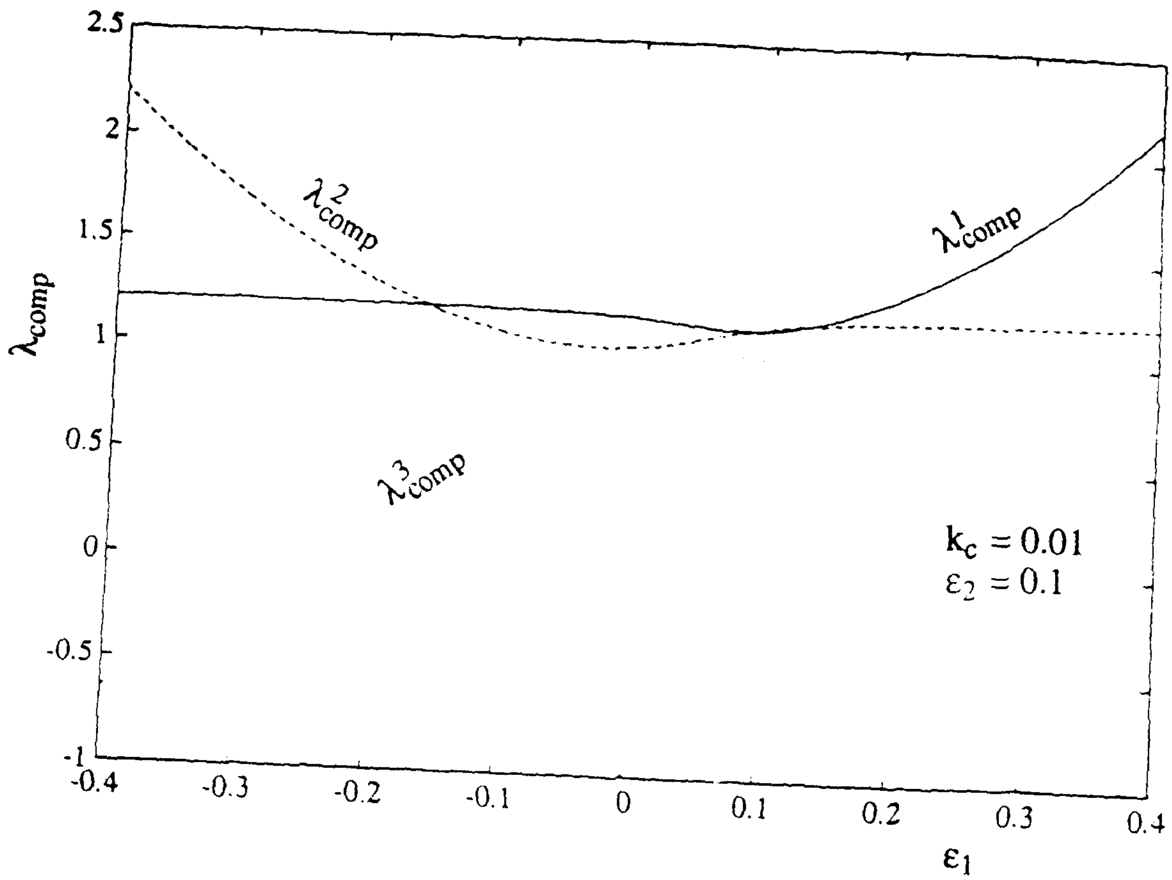

Figure 9. Composite eigenvalues in case of weak coupling; $k_c = 0.01$, $k_t = 1$, $\varepsilon_2 = 0.1$, $m = 1$, $r = 1$.

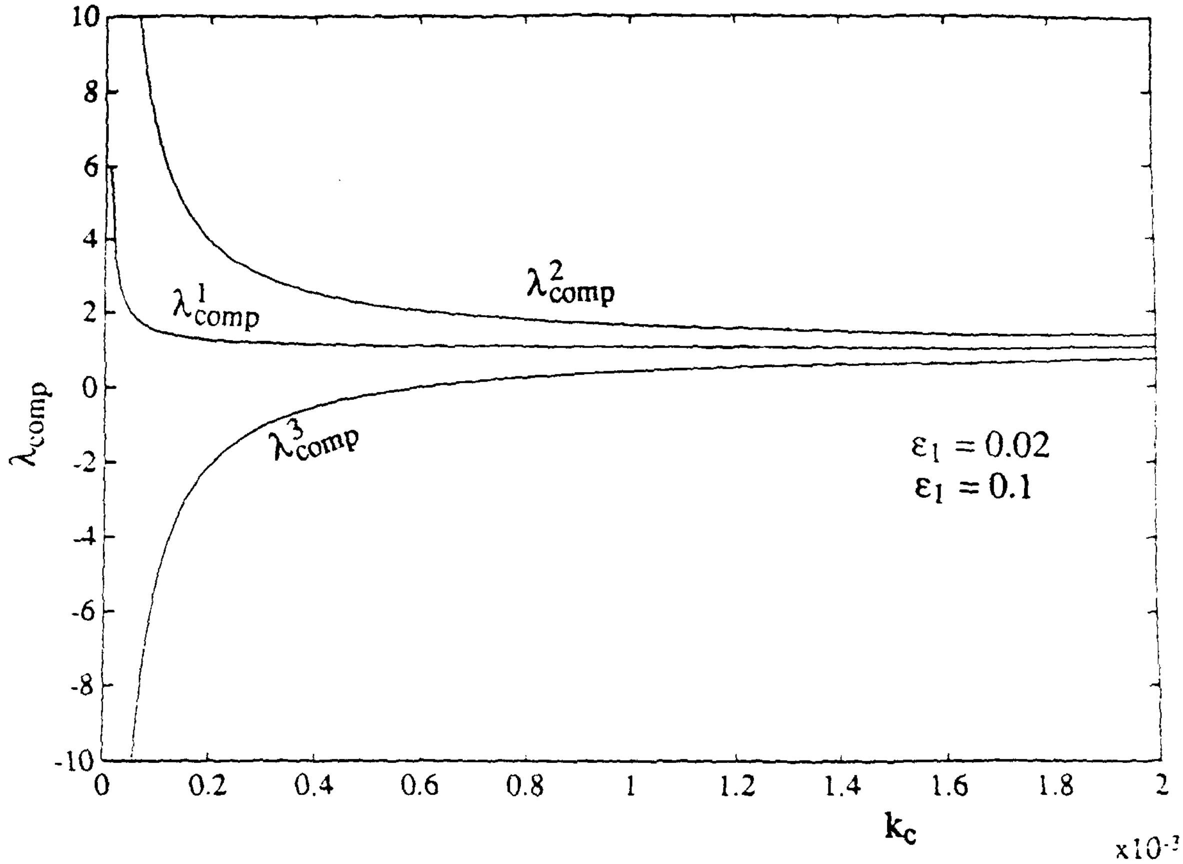

Figure 10. Behavior of the composite eigenvalues as a function of the coupling parameter k_c;

$k_t = 1, \varepsilon_1 = 0.02, \varepsilon_2 = 0.1, m = 1, r = 1.$

APPENDIX 3

On the Modal Stability of Imperfect Cyclic Systems

-O.D.I. Nowkah, D. Afalobi, F. M. Damra

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ON THE MODAL STABILITY OF IMPERFECT CYCLIC SYSTEMS

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I. Introduction

An important subject in the dynamics and control of structural systems is the behavior of structures under transient or steady state excitations. In this work, we examine the stability of the geometric form of the spatial configuration of structural systems when the structural parameters are subject to small perturbations, and the implications of this instability for frequency response. We show that circularly configured systems which nominally have cyclic symmetry exhibit complicated topological behavior when small perturbations are impressed on them. We further show that the frequency response of a perturbed cyclic system depends considerably on the form of perturbation. On the other hand, a rectilinear configuration of nearly identical subsystems does not ehibit modal instability. Usually, both kinds of systems are implicitly assumed to undergo similar qualitative behavior under a small perturbation whereas, in fact, the cyclic configuration exhibits a very stange behavior, [1].

The distinction between the behavior of cyclic and rectilinear configurations under a perturbation is important because many engineering structures are composed of identical substructures which are replicated either in a *uni-axial* chain, or in a closed *cyclic* formation where modal control is of interest. Examples of the former case of periodicity occur in structures such as space platforms and bridges, which have an obvious periodicity of the uni-axial kind. An example of cyclic periodic systems is a turbine rotor, which consists of a set of nominally identical blades mounted on a central hub, and often referred to as a "bladed disk assembly" [2]. When all the blades are truly identical, then the system is referred to in the literature as a *tuned* bladed disk assembly. Practical realities of manufacturing processes preclude the existence of exact uniformity among all the blades. When residual differences from one blade to another—no matter how small—are accounted for in the theoretical model, the assembly is then termed a *mistuned* bladed disk.

Our primary focus in this investigation is on bladed disk assemblies. However, since we approach the problem from a generalized viewpoint, the conclusions to be drawn will be of relevance to other periodic systems. Therefore, in the sequel, we borrow the 'tuned' and 'mistuned' terminology from the bladed disk literature, and apply it to repetitive systems having cyclic or uniaxial periodicity. Thus, in a tuned periodic system, the nominal periodicity is preserved, whereas it is destroyed in a mistuned system.

If we examine the system matrices of the linear and cyclic chains, we observe a fundamental difference in forms. The dynamical matrix of the linear chain is usually banded. Banded matrices are frequently encountered in structural dynamics. A special form of banded matrices that is of interest here is the tri-diagonal form $a_{ij}=0, |i-j|> 1$. On the other hand, the system matrix of a cyclic chain has a circulant submatrix, or is entirely circulant or block circulant [3]. Circulant matrices usually arise in the study of circular systems. They have interesting properties that set them apart from matrices of other forms [4]. We note that all circulants commute under multiplication, and are diagonalizable by the fourier matrix. One of the most important consequences of the foregoing is that the cyclic chain has a *series* of degenerate eigenvalues, whereas the eigenvalues of the uniaxial chain are all simple.

We know that a tuned circulant matrix, having a multitude of degenerate eigenvalues, lies on a bifurcation set [5]. Thus, the reduction of such matrices to Jordan normal form is an unstable operation [6]. Consequently, if a non-singular deformation due to mistuning is applied to a circulant matrix, then some of the eigenvectors will undergo rapid re-alignment, if the mistuning leads to a crossing of the bifurcation set. If however, no crossing of the bifurcation set takes place, then the tuned system's eigenvectors will be very stable, preserving their alignment under small perturbations. In contrast, the eigenvectors of a tuned banded matrix, being analytically dependent on parameters, are not generally disoriented by mistuning until the eigenvalues are pathologically close [7].

If one examines the literature in structural dynamics, it is observed that some unusual behavior has been reported in the study of perturbed cyclic systems. This has been the case in various studies of rings [8], circular saws [9], and other cyclic structures [10]. But that such anomalous behavior is due to a "geometric instability" inherent in the *cyclicity* of the tuned system has not been previously established in the literature, to our knowledge. Indeed, it is often assumed (see, for instance, [11]) that the linear and cyclic chains would undergo the same qualitative behavior under slight parameter perturbations so that small order perturbations of the system matrix will lead to no more than small order differences in the system response relative to the unperturbed case, if the system has "strong coupling".

In this paper, we show that such an assumption regarding qualitative behavior does not actually hold in the case of cyclic systems; that cyclic systems exhibit a peculiarity of their own under parameter perturbation; that, although a certain amount of mistuning may produce little difference relative to the tuned datum in one case, a considerable change could be induced if a slightly different kind of mistuning is applied to the same cyclic system; that such apparently erratic behavior arises in cyclic system, even when the system has "strong" coupling. In carrying out this work, we borrow from certain developments in differential topology specifically, from Arnold's monumental work in singularity theory [6, 12-16].

II. Topological Dynamics of Quadratic Systems

In mistuned dynamical systems, a major concern is to understand which specific kinds of mistuning parameters, or combinations thereof, lead to unacceptably high amplitude ratios. In this section, we give an indication of the taxonomy of the different consequences of mistuning in the hope of isolating those that lead to high ratios.

Consider a mechanical system under small oscillations with kinetic and potential energies given by:

$$T = \tfrac{1}{2}\dot{x}^{*} M\dot{x} > 0,\ U = \tfrac{1}{2}x^{*} Kx > 0;\ \ \dot{x}, x \neq 0. \tag{2.1}$$

Under the influence of a forcing function $f(t)$, (2.1) produces the following equations of motion by application of Lagrange's formula:

$$M\ddot{x} + Kx = f;\ \ x, f \in \mathbb{C}^n \tag{2.2}$$

where M and K are symmetric and positive definite. A theorem in linear algebra shows that there exists some non-singular transformation matrix P such that:

$$P^T MP = I, \text{ and } P^T KP = \Lambda \tag{2.3}$$

where Λ is a diagonal matrix of eigenvalues whose elements satisfy the equation:

$$\det(M - \lambda K) = 0 \tag{2.4}$$

Consequently, by putting

$$x = Pq, \tag{2.5}$$

substituting for q in (2.1), and premultiplying every term of the resultant equation by P^T, we obtain a new equation set:

$$\ddot{q} + \Lambda q = f', \tag{2.6}$$

where $f' = P^T f$. Hence:

$$\ddot{q}_i + \lambda_i q_i = f'_i \text{ for } i = 1, 2, \cdots, n. \tag{2.7}$$

Systems which can be reduced to the above form are called quadratic systems. They are called quadratic *cyclic* systems if, in addition, M and K are cyclic or circulant matrices. Our basic aim is to determine the nature of the changes in the dynamical properties of a quadratic system of a given order, under random differential perturbations in M and/or K. Central to this investigation are the topological concepts of structural stability and genericity.

Let N be a set with a topology and an equivalence relation e. An element $x \in N$ is stable (relative to e) if the e-equivalence class of x contains a neighborhood of x.

A property P of elements of N is generic if the set of all $x \in N$ satisfying P contains a subset A which is a countable intersection of open dense sets [17].

Genericity is important in our context because a generic system will in effect display a "typical" behavior. More concretely if a given generic system gives a certain frequency response, all systems produced by differential parameter perturbations about the nominal system will also produce frequency response curves that are not only slight perturbations of the original nominal response but also geometrically (isomorphic) equivalent to it. Such systems are called versal deformations of the nominal system [14]. A versal deformation of a system is a normal form to which it is possible to reduce not only a suitable representation of a nominal system, but also the representation of all nearby systems such that the reduction transformation depends smoothly on parameters. The key to establishing versality, and hence genericity, is the topological concept of transversality.

Let $N \subset M$ be a smooth submanifold of the manifold M. Consider a smooth mapping $f: \Gamma \to M$ of the parameter space Γ into M; and let μ be a *point in* Γ *such* that $f(\mu) \in N$.

The mapping f is transversal to N at μ if the tangent space to M at $f(\mu)$ is the sum:

$$TM_{f(\mu)} = f_{*}\, T\Gamma_{\mu} + TN_{f(\mu)}$$

Consequently, two manifolds intersect transversally if either they do not intersect at all or intersect properly such that perturbations of the manifolds will neither remove the intersection nor alter the type of intersection.

Lemma 2.1, see ref [14].

A deformation $f(\mu)$ *is versal if and only if the mapping* $f: \Gamma \to M$ *is transversal to the orbit of* f *at* $\mu = 0$.

The abov^ result is crucially important because:

(i) It classifies from the set of all perturbations of a given nominal system, those that do not lead to radically different dynamical properties from the nominal.

(ii) It separates the "good" from the "bad" perturbations and hence enables us to concentrate our study on the bad perturbations. Let Q denote the family of all real quadratic systems in $\mathbb{R}^n$. The set Q has the structure of a vector space of dimension $\frac{1}{2}(n[n+1])$. It can be shown that Q also has the structure of a *differentiable manifold* [13].

Let Q_ν denote the set of quadratic systems having ν_2 eigenvalues of multiplicity 2, ν_3 eigenvalues of multiplicity 3 etc. Q_ν is called the degenerate subfamily of Q.

Theorem 2.1, see ref [13].
The transformation $f:\Gamma \rightarrow Q$ *is transversal to* Q_ν.

Consequently, a generic family of quadratic sys..ns of a given order is given by a transformation, f, of the space of parameters Γ into the space of all quadratic systems Q, such that f is transversal to the space of all degenerate quadratic systems Q_ν.

Hence Q_ν is the degenerate (bad) set and Q/Q_ν 's the generic set. Observe that Q/Q_ν and Q_ν are transversal. Consequently, the fundamental group of the space of generic real quadratic systems is isomorphic to the manifold of systems without degenerate eigenvalues.

The above discussion leads inevitably to the following conclusions:

(i) Radical changes in the dynamical properties of a nominal system occurs under perturbations, when the perturbations take the system across the boundary from Q/Q_ν to Q_ν and vice-versa.

(ii) Q_ν is a smooth semi-algebraic submanifold of Q, and can therefore be stratified into distinct fiber bundles [14]. By a bundle, we mean the set of all systems which differ only by the exact values of their eigenvalues; but for which the number of distinct eigenvalues as well as the respective orders of the degenerate eigenvalues are the same. Within the degenerate set, Q_v, the crossing from one bundle to another can also lead to radical dynamical changes. Each bundle is represented by a specific Jordan block of a certain order. Note that each bundle is also transversal to Q.

Theorem 2.2, ref [14].

Q_v is a finite union of smooth sub-manifolds with codimension satisfying Codim $Q_v \geq 2$.

Theorem 2.2 has the following implications:

(i) Q/Q_v is topologically path connected. This means that by smooth parameter variations, provided that the number of variable parameters is less than the codimension of Q_v, it is possible to smoothly pass from one member of Q/Q_v to another without reaching any singularity; that is, without encountering any member of Q_v. Such parameter variations will typically not lead to radical dynamical changes in Q/Q_v.

(ii) Because codim $Q_v \geq 2$, it follows that a generic one-parameter family of quadratic systems cannot contain any degenerate subfamilies. Therefore under one-parameter deformations of a generic family, some eigenvalue pairs may approach each other but cannot be coincident (i.e. cannot collide). After approaching each other, they must veer away rapidly, giving rise to the so-called eigenvalue loci-veering phenomenon [18], under one-parameter deformations of generic families. This offers a theoretical explanation for the eigenloci veering phenomenon which has been observed in perturbed periodic systems without a corresponding phenomenological base [18, 19]. Furthermore, this phenomenon holds provided the system has a quadratic structure, irrespective of whether the model arose from a continuous or discrete structural system [20].

This rapid eigenloci veering can, under the right conditions, produce the mode localization phenomenon [18]. Since the dynamical properties of any linear constant-coefficient system are totally determined by its eigen-structure (eigenvalues and eigenvectors), and since the eigenvalues are continuous functions of the matrix elements, it follows that radical changes in the dynamical properties of a given system under differential parameter perturbations ensue principally from a large disorientation between the eigenvectors of the tuned (unperturbed) and mistuned (perturbed) systems. We study, in Section IV, the variation of eigenvectors of generic families under differential random parameter perturbations.

III. Bounds on Amplitude Ratios

Consider, again, the equation set for the dynamics of quadratic systems:

$$M\ddot{x}_0 + Kx_0 = f, \tag{3.1}$$

where M and K are positive definite matrices. For tuned cyclic systems, M and K have the additional property of being circulant. Taking the Laplace transform of (3.2) under zero initial conditions, gives:

$$(Ms^2 + K)X_0(s) = F(s), \tag{3.2}$$

or

$$A(s) \cdot X_0(s) = F(s) \tag{3.3}$$

where $A = Ms^2 + K$. Suppressing s in all subsequent calculations leads to:

$$X_0 = A^{-1} \cdot F. \tag{3.4}$$

The positive definite nature of M and K guarantees that A^{-1} exists for all s on the Nyquist contour. Under normal operations of the system, suppose A varies to $A + \Delta A := A_e$. Let X_0 then change to $X_0 + \Delta X := X_e$. Then, for the same excitation force as in the tuned state,

$$X_e = (A + \Delta A)^{-1} \cdot F. \tag{3.5}$$

The physical nature of the system guarantees that $A + \Delta A$ will always remain symmetric but not *necessarily* circulant since a true mistuning destroys cyclicity. Equation (3.5) can be rewritten as:

$$X_e = (A + \Delta A)^{-1} \cdot F = (I + A^{-1}\Delta A)^{-1} \cdot A^{-1}F. \tag{3.6}$$

Substituting (3.4) into (3.6) gives:

$$X_e = (I + A^{-1}\Delta A)^{-1} \cdot X_0. \tag{3.7}$$

Normally ΔA will be a differential perturbation of A, so that:

$$\rho(A^{-1}\Delta A) < 1,$$

where $\rho(\cdot)$ is the spectral radius of $(\cdot)$. Hence

$$(I + A^{-1}\Delta A)^{-1} = \sum_{k=0}^{\infty} (-1)^k (A^{-1}\Delta A)^k. \tag{3.8}$$

Substituting (3.8) into (3.7) gives:

$$X_e = \sum_{k=0}^{\infty} (-1)^k (A^{-1}\Delta A)^k \cdot X_0. \tag{3.9}$$

Taking norms in (3.9) gives:

$$\|X_e\| = \| \sum_{k=0}^{\infty} (-1)^k (A^{-1}\Delta A)^k X_0 \|$$

$$\leq \sum_{k=0}^{\infty} \| A^{-1}\Delta A \|^k \cdot \|X_0\|. \tag{3.10}$$

Let $\|A^{-1}\Delta A\| = r$. Because ΔA is a differential perturbation of A, it follows that $r < 1$. Hence:

$$\| X_e \| \leq \|X_0\| \sum_{k=0}^{\infty} r^k = \|X_0\| \left\{ 1 + r + r^2 + \cdots + r^k \right\}$$

$$\leq \frac{\|X_0\|}{1-r}, \text{ since } r < 1. \tag{3.11}$$

Or:

$$\frac{\|X_e\|}{\|X_0\|} \leq \frac{1}{1-r} = \frac{1}{1 - \|A^{-1}\Delta A\|}. \tag{3.12}$$

Write

$$A = D + C = D(I + D^{-1}C) \tag{3.13}$$

where D is a diagonal matrix of the uncoupled system dynamic matrix and C is the relative coupling dynamic matrix, such that the minimum eigenvalue of $D^{-1}C$ at any frequency gives the coupling index of the system at that frequency [21]. If the norms in (3.12) are H^{∞}- norms, then, over the frequency interval Ω:

$$\underset{\omega\in\Omega}{\text{ess.sup}}\left\{\frac{\|X_e(\omega)\|_\infty}{\|X_0(\omega)\|_\infty}\right\} \le \underset{\omega\in\Omega}{\text{ess.sup}}\left\{\frac{1}{1-\left|\dfrac{\sigma_{\max}\Delta A(\omega)}{\sigma_{\min}A(\omega)}\right|}\right\} \tag{3.14}$$

where $\sigma_{\max}(\cdot)$ and $\sigma_{\min}(\cdot)$ correspond to maximum and minimum singular values of $(\cdot)$ respectively. Note that all the matrices and vectors considered above are functions of frequency $s = i\omega$.

Because A is symmetric it follows from (3.13) that:

$$\begin{aligned}\sigma_{\min}(A) &= \sigma_{\min}(D[I + D^{-1}C]) \\ &= \lambda_{\min}(D)\cdot\lambda_{\min}(I + D^{-1}C), \\ &= d_{\min}[1 + \lambda_{\min}(D^{-1}C)]\end{aligned} \tag{3.15}$$

by the eigenvalue shift theorem, where $d_{\min}$ is the minimum eigenvalue of D. Let $d_{\min} = a$, and $\lambda_{\min}(D^{-1}C) = \lambda_0$. At any frequency ω, let $\dfrac{\|X_e(\omega)\|_\infty}{\|X_0(\omega)\|_\infty} = \Pi(\omega)$. Then (3.14) reduces to:

$$\delta_e = \underset{\omega\in\Omega}{\text{ess.sup}}\ \Pi(\omega) \le \frac{1}{1-\underset{\omega\in\Omega}{\text{ess.sup}}\left\{\left|\dfrac{\sigma_{\max}\Delta A(\omega)}{a(\omega)(1+\lambda_0(\omega))}\right|\right\}} \tag{3.16}$$

where Ω is a frequency interval of interest. In some cases it is possible to define Ω by the semi-open interval $\Omega = [\,0\ \ \infty)$. Here λ_0 is called the coupling index of the system. The system is decoupled when $\lambda_0 = 0$. It is weakly coupled if $\lambda_0 < 1$, and is strongly coupled if $\lambda_0 \geq 1$. In general, $0 \leq \lambda_0 \leq \infty$. Observe that $\lambda_0(k, \omega)$ is a function of both the structural coupling k, and frequency ω. Inequality (3.16) leads to the following conclusions:

(i) The mistuned to tuned amplitude ratio is determined by the maximum peak of the mistuning strength $\sigma_{max}\Delta A(\omega)$, the minimum strength of the weakest link in the system $a(\omega)$, and the minimum peak of the coupling index (strength) $\lambda_0(\omega)$.

(ii) A variation in rigidity (coupling) affects the ratio of (3.16) monotonically for fixed values of $\sigma_{max}(\Delta A)$ and a. This is because at any given frequency, λ_0 varies continuously and monotonically as the coupling is varied [13].

(iii) A reduction in a caused by a reduction of mass of the blades, and/or more flexible blades, increases the ratio (3.16) monotonically. More specifically, at any frequency when $\lambda_0 \to 0$, from (3.16):

$$\delta_e \leq \operatorname*{ess.sup}_{\omega \in \Omega} \left\{ \left| \frac{a(\omega)}{a(\omega) - \sigma_{max}\Delta A(\omega)} \right| \right\} > 1, \text{ for } \sigma_{max}\Delta A(\omega) > 0, \ \forall\ \omega \in \Omega.$$

Hence under weak coupling across the frequency interval, the amplitude ratio depends entirely on the relationship between the frequency response of the mistuning strength and that of the strength of the weakest blade in the assembly. Under these conditions, the maximum amplitude ratio will arise from the blade with the worst mistune [22].

IV. Eigenvector Rotations

In section II, we showed that generic systems Q / Q_v will typically have distinct eigenvalues, while degenerate systems Q_v will typically have repeated eigenvalues. To study eigenvector perturbations for generic systems, regular analytical methods will work, while for eigenvector variations in the system Q_v we require singular perturbations [23]. Let $A \in \mathbb{C}^{n \times n}$ be the dynamic matrix arising from any system $Q_r \in Q/Q_\nu$. Let Γ represent the parameter space and let $\mu \in \Gamma$ be a p-dimensional parameter vector. If Codim $Q_\nu \geq r$, then for any $\mu \in \Gamma \in \mathbf{R}^p$, where $p<r$, differential parameter variations in $A(\mu)$ will not lead to eigenvalue degeneracies. Thus, if the eigenvalues of $A(\mu)$, given by $\lambda_1(\mu), \lambda_2(\mu), \cdots, \lambda_n(\mu)$, are distinct when $\mu=0$ they will continue to remain distinct when μ is small, by continuity arguments.

Let

$$A(\delta\mu) = A(0) + \delta A, \tag{4.1}$$

where:

$$\delta A = \delta\mu_k \cdot \frac{\partial}{\partial\mu_k} A(\mu) \Big|_{\mu=0} \tag{4.2}$$

δA can be expanded in Taylor series form as: $\delta A = \Delta A$ + higher terms dependent on μ. To a first order approximation we can write the perturbed matrix as:

$$A = A_0 + \Delta A. \tag{4.3}$$

Write

$$A_0 = U \Lambda U^{-1} \tag{4.4}$$

where U is the modal matrix of A_0, and $V^* = U^{-1}$ where:

$$U = [u_1, u_2, \cdots, u_n]$$

and

$$V^* = [v_1^* \; v_2^*, \cdots, {}_n^*]^*$$

with

$$\Lambda = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n).$$

where $(\cdot)^*$ is the complex conjugate transpose of $(\cdot)$.

Since A_0 is also generic, we can write the perturbed modal expression as:

$$A_0 + \Delta A = [U + \Delta U][\Lambda + \Delta\Lambda][U + \Delta U]^{-1} \tag{4.5}$$

where ΔU is the perturbation in U resulting from ΔA while $\Delta\Lambda$ is the corresponding perturbation in Λ resulting from ΔA. Under eigenvector normalization, $\| u_i \| = 1$, and $\| u_i + \Delta u_i \| = 1$. Equation (4.5) can be solved as:

$$A_0 U + \Delta A U + A_0 \Delta U = U\Lambda + U\Delta\Lambda + \Delta U \Lambda \tag{4.6}$$

where we neglect second order terms like $\Delta U \Delta\Lambda$ and $\Delta A \Delta U$ [24]. As a measure of the eigenvector variations, we begin by writing Δu_i as a linear combination of all the eigenvectors since the eigenvectors span the whole n-dimensional space. Thus:

$$\Delta u_i = \sum_{j=1}^{n} l_{ji} \, u_j \tag{4.7}$$

Or:

$$\Delta U = UL. \tag{4.8}$$

Now solving for $\Delta\Lambda$ in (4.6) gives:

$$\Delta\Lambda = U^{-1} A_0 U + U^{-1} \Delta A U + U^{-1} A_0 \, \Delta U - \Lambda - U^{-1} \Delta U \Lambda. \tag{4.9}$$

Observe that $U^{-1} A_0 U - \Lambda = 0$, so that

$$\Delta\Lambda = U^{-1} \Delta A U + \Lambda L - L\Lambda. \tag{4.10}$$

Notice that the diagonal elements of $(\Lambda L - L\Lambda)$ are zero. Hence:

$$\Delta\lambda_i = [\, U^{-1} \Delta A U]_{ii} = v_i^* \, \Delta A u_i.$$

To solve for ΔU, we need L. The off-diagonal elements of L are given by Skelton [24]

$$l_{ji} = (\lambda_j - \lambda_i)^{-1} \, v_i^* \, \Delta A u_j \quad \text{for } i \neq j, \; i,j = 1, 2 \cdots n \, ,$$

or:

$$l_{ij} = (\lambda_i - \lambda_j)^{-1} \, v_j^* \, \Delta A u_i \quad \text{for } i \neq j, \; i,j = 1, 2 \cdots n.$$

To determine l_{ii}, observe that the constraint equation $\| u_i + \Delta u_i \| = 1$ contains l_{ii}. Thus

$$\| u_i + \Delta u_i \| = \{ < u_i + \Delta u_i , \, u_i + \Delta u_i > \}^{1/2} = 1. \tag{4.12}$$

Or:

$$u_i^* u_i + 2u_i^* \Delta u_i + \Delta u_i^* \Delta u_i = 1. \tag{4.13}$$

But $u_i^* u_i = 1$, so that

$$2u_i^* \Delta u_i + \Delta u_i^* \Delta u_i = 0.$$

Therefore:

$$\Delta u_i = \sum_{j=1}^{n} l_{ji} u_j = \sum_{\substack{j=1 \\ j \neq i}}^{n} l_{ji} u_j + l_{ii} u_i \tag{4.14}$$

$$= x_i + l_{ii} u_i$$

where:

$$x_i = \sum_{\substack{j=1 \\ j \neq i}}^{n} l_{ji} u_j. \tag{4.15}$$

Thus:

$$l_{ii}^2 + (2 + 2u_i^* x_i) l_{ii} + (2u_i^* x_i + x_i^* x_i) = 0 \tag{4.16}$$

Letting:

$$z_i = u_i^* x_i$$

and

$$y_i = x_i^* x_i,$$

gives (on accepting the positive solution of the quadratic):

$$l_{ii} = -(1 + z_i) + (1 + z_i^2 - y_i)^{1/2}. \tag{4.17}$$

Since the eigenvectors u_i and $u_i + \Delta u_i$ can be normalized to unity and since each vector is represented by a magnitude m_i and an angle θ_i, the natural measure of modal variations is θ_i since $m_i \equiv 1$ after normalization. Knowing all the elements of L, we can now determine θ_i as:

$$<u_i, u_i + \Delta u_i> = \| u_i \| \; \| u_i + \Delta u_i \| \cos \theta_i . \tag{4.18}$$

But $\| u_i \| = \| u_i + \Delta u_i \| = 1$.

Hence:

$$\begin{aligned} \cos \theta_i &= <u_i, u_i> + <u_i, \Delta u_i> \\ &= 1 + u_i^*(x_i + l_{ii}\, u_i) \\ &= 1 + u_i^* x_i + l_{ii} \\ &= (1 + z_i^2 - y_i)^{1/2}, \quad 0 \le \theta_i \le \pi/2. \end{aligned} \tag{4.19}$$

Consequently for the occurrence of no vector rotation under parameter variations, we require:

$$z_i^2 - y_i = 0 \tag{4.20}$$

Or:

$$x_i^* u_i\, u_i^* x_i - x_i^* x_i = x_i^* (u_i u_i^* - I) x_i = 0 \tag{4.21}$$

This implies x_i belongs to the null space of $(u_i u_i^* - I)$, that is:

$$(u_i u_i^* - I) \sum_{\substack{j=1 \\ j \neq i}}^{n} lji\; u_j = 0 \tag{4.22}$$

where:

$$l_{ji} = (\lambda_i - \lambda_j)^{-1}\, v_j^*\, \Delta A\, u_i \qquad i \neq j$$

The nearer the expression (4.20) is to zero, the less the corresponding eigenvector rotation under the given perturbation. Let

$$\alpha_i = \sum_{\substack{j=i \\ j \neq i}}^{n} l_{ji}\, u_j^*\, (u_i u_i^* - I) \sum_{\substack{j=i \\ j \neq i}}^{n} l_{ji}\, u_j, \quad i = 1, 2 \cdots n.$$

Then $\max_i \{\alpha_i\}$ gives the eigenvector with maximum rotation.

The conclusions are the following:

a) If the separation between the eigenvalues is very large, (i.e. $(\lambda_i - \lambda_j)$ is very large for all i,j), then l_{ji} is relatively small and eigenvector rotation will be correspondingly small.

b) If $v_j^* \Delta A u_i \simeq 0$, then eigenvector rotation will also be relatively small, provided $l_{ji} \neq \infty$.

For example, if A_0 is Hermitian as is the case in all quadratic systems, and $\Delta A = \alpha I, \alpha \in \mathbb{C}$, then

$$v_j^* \Delta A u_i \equiv 0, \ \forall \ i,j = 1,2, \cdots n.$$

Thus, identical increases or decreases in the diagonal elements of a quadratic system will not produce unexpected amplitude excursions [25] because it cannot produce eigenvalue splittings in formerly degenerate families. Therefore, such perturbation cannot take a system either across the boundary of the bifurcation set or across different bundles of Q_v. Geometrically, this implies that degenerate eigenvalues in systems belonging to a bundle in Q_v cannot be lifted by perturbations that leave the perturbed system in the same bundle of Q_v. Indeed, define the eigenvector sensitivity matrix of a quadratic system as

$$S = \Delta U U^{-1} = L,$$

from eqn. (4.8). Defining the eigenvector sensitivity metric measure by

$$S_F = \| S \|_2^2 = \sum_{\substack{j=1 \\ i=1}}^{n} l_{ji}^2$$

where S_F is the Frobenius norm of S shows that the maximum eigenvector sensitivity is obtained at the positions of minimum eigenvalue separation, which is not difficult to compute. Alternatively, $(S_F)_{max}$ also corresponds to the position of maximum angular rotation between the tuned and mistuned system eigenvectors. This condition is evidenced by strong eigenloci deformations.

If $A(\omega)$ is a frequency response matrix arising from a generic system, the eigenvalues $\lambda_i(\omega)$ and eigenvectors $u_i(\omega)$ are also continuous functions of frequency. We can therefore plot the frequency response functions $S_F(\omega)$ to determine the frequencies at which maximum deformations take place.

V. Examples

To illustrate the theory so far developed we consider two examples. The first is an interconnected linear chain of oscillators. This has been studied by Arnold [13] and more recently by Pierre [18].

Example 1: *Mode Localization in Generic Periodic Systems.*

Consider a coupled pendulum, as shown in Fig 1, with identical masses but of different lengths l_1 and l_2, where l_2 is a perturbation of l_1, i.e., $l_2 = (l_1 + \Delta l_1)$. If we put $l_1 = l$, then the kinetic energy is given by

$$T = \frac{1}{2} m[l^2 \theta_1^2 + (l + \Delta l)^2 \theta_2^2] \tag{5.1}$$

while the potential energy is given by

$$U = ml\frac{\theta_1^2}{2} + m(l + \Delta l)\frac{\theta_2^2}{2} + \frac{k}{2}(\theta_1 - \theta_2)^2. \tag{5.2}$$

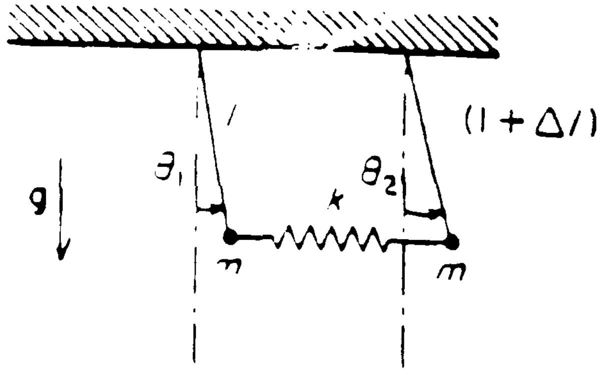

Fig. 1. *Two coupled oscillators.*

Under unit gravitational force, application of Lagrange's equations results in the equation of motion:

$$M\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} + \begin{bmatrix}K\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} = 0 \tag{5.3}$$

where

$$M = \begin{bmatrix} ml^2 & 0 \\ 0 & m(l+\Delta l)^2 \end{bmatrix} \tag{5.4}$$

and

$$K = \begin{bmatrix} ml + k & -k \\ -k & m(l+\Delta l)^2 + k \end{bmatrix}. \tag{5.5}$$

The dynamic matrix for the above system is given by

$$A(\omega) = \begin{bmatrix} ml + k - \omega^2 ml^2 & -k \\ -k & m(l+\Delta l) + k - \omega_m^2 (l+\Delta l)^2 \end{bmatrix}. \tag{5.6}$$

Rewrite $A(\omega)$ as:

$$A(\omega) = \begin{bmatrix} a & -k \\ -k & b \end{bmatrix}. \tag{5.7}$$

The characteristic equation of $A(\omega)$ is given by

$$\lambda^2 - (a+b)\lambda + (ab - k^2) = 0$$

Both M and K are symmetric and positive definite. The eigenvalues of $A(\;)$ are:

$$\lambda_{1,2} = \frac{[(a+b) \pm \sqrt{(a-b)^2 + 4k^2}\,]}{2}. \tag{5.8}$$

Note that $\lambda_{1,2}$ cannot be degenerate. Thus under one-parameter deformations, the eigenvalues can deform but cannot collide.

Indeed,

$$\frac{\partial \lambda_1}{\partial a} = \frac{1}{2}\left[1 + \frac{a-b}{\sqrt{(a-b)^2 + 4k^2}}\right] \tag{5.9}$$

and

$$\frac{\partial \lambda_2}{\partial a} = \frac{1}{2}\left[1 - \frac{a-b}{\sqrt{(a-b)^2 + 4k^2}}\right]. \tag{5.10}$$

Hence:

$$\frac{\partial \lambda_1}{\partial a} + \frac{\partial \lambda_2}{\partial a} = 1 \tag{5.11}$$

$\left|\frac{\partial \lambda_i}{\partial a}\right| = \frac{1}{2}$ when $a=b$.

The distance between the eigenvalues is given by:

$$d_\lambda = |\lambda_1 - \lambda_2| = \sqrt{(a-b)^2 + 4k^2} \tag{5.12}$$

which assumes its minimum value of $2k$ when $a=b$ or when $\left|\partial\lambda_i/\partial a\right| = ½$ This represents the tuned state of the linear chain. For a fixed mistuning value $(a-b)$, d_λ depends essentially on k. If $(a-b)$ is small, it is clear that $S_F \to \infty$ as $k \to 0$. The modal matrix of the chain is given by

$$U = \begin{bmatrix} -1 & -1 \\ \dfrac{(a-b) - \sqrt{(a-b)^2 + 4k^2}}{2k} & \dfrac{(a-b) + \sqrt{(a-b)^2 + 4k^2}}{2k} \end{bmatrix}. \tag{5.13}$$

Observe that $u_i^* \, u_j \equiv 0, \;\; \forall \; k,a,b$. Under tuned conditions, $a=b$, then

$$U_t = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \tag{5.14}$$

However, consider the very interesting situation when the mistuning to coupling ratio is rather large. That is to say:

$$\frac{(a-b)}{k} > 1.$$

Then $(a-b)^2 \gg k^2$, and k^2 becomes negligible in the eigenvector expressions. Expanding the term under the radicals and neglecting second and higher order terms gives:

$$[(a-b)^2+k^2]^{1/2}=(a-b)\left\{1+\left[\frac{k}{a-b}\right]^2\right\}^{1/2}$$

$$=(a-b)[1+\frac{1}{2}\left[\frac{k}{a-b}\right]^2+\cdots \tag{5.15}$$

In this case the modal matrix reduces to:

$$U_e=\begin{bmatrix} -1 & -1 \\ -\dfrac{k}{a-b} & \dfrac{a-b}{k} \end{bmatrix}. \tag{5.16}$$

An energy exchange now takes place. The second component of the first mode becomes vanishingly small while the corresponding component of the second mode becomes extremely large. This is an extreme case of classical vibration absorber, and is the *mode localization* phenomenon. We therefore conclude that mode localization (or extreme energy exchange) will occur in a generic system under one-parameter deformations if the following conditions are satisfied:

- at any frequency ω where the system is almost decoupled, i.e., $\lambda(\omega)_0 \to 0$. (Note that $\lambda(\omega)_0 \to 0$ as $k \to 0$).
- when the mistuning to coupling ratio $\dfrac{a-b}{k} \gg 1$.

At the localization stage the eigenvalue and eigenvector sensitivities take on their maximum values, i.e. both $\| \Delta\Lambda\, \Lambda^{-1} \|_F^2$ and $\| \Delta U\, U^{-1} \|_F^2$ have their maximum values. Localized modes always produce:

$$\delta_e=\frac{\| x_e \|_\infty}{\| x_0 \|_\infty} \gg 1. \tag{5.17}$$

Example 2: *Cyclic Systems.*

Consider three identical masses, m, arranged in a ring structure and interconnected by identical springs k_c. Assume that all the masses are hinged to the ground by torsional springs of strength k_t, and that the radius of the ring is r; as shown in Fig. 2.

The basic equations of motion of this "ring" is

$$M\ddot{x} + Kx = f \tag{5.18}$$

where

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix}, \quad K = \begin{bmatrix} 2k_c + \dfrac{k_t}{r} & -k_c & -k_c \\ -k_c & 2k_c + \dfrac{k_t}{r} & -k_c \\ -k_c & -k_c & 2k_c + \dfrac{k_t}{r} \end{bmatrix}. \tag{5.19}$$

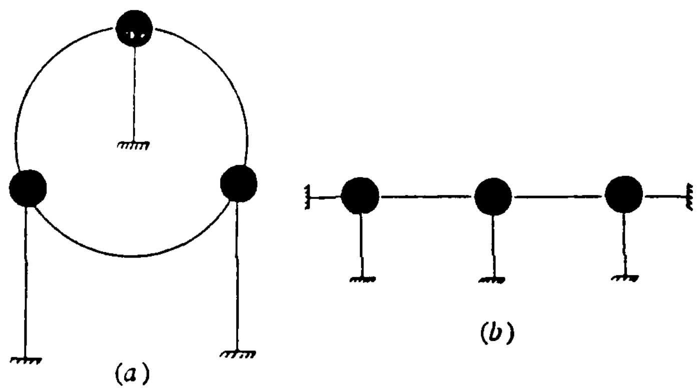

Fig. 2. *Models of (a) the cyclic chain, (b) the linear chain with three degress of freedom.*

Using group theoretic arguments [13], we can easily deduce that the above system has degenerate eigenvalues occurring as doublets, by cyclicity of the corresponding system matrices. Consequently, every quadratic cyclic system $Q_c \subset Q_V$. Furthermore all perturbations of the above system preserving the cyclic structure, leaves the modal geometry invariant [3, 25]. Indeed the eigenvalues of the above system are given as:

$$\lambda_1 = \frac{k_t}{mr}, \quad \lambda_2 = \frac{k_t}{mr} + \frac{3k_c}{m}, \quad \lambda_3 = \frac{k_t}{mr} + \frac{3k_c}{m}. \tag{5.20}$$

Write the dynamic matrix of this system as:

$$A_0(\omega) = \begin{bmatrix} a & -b & -b \\ -b & a & -b \\ -b & -b & a \end{bmatrix} \tag{5.21}$$

where

$$a = 2k_c + k_t - \omega^2 m, \quad b = k_c.$$

We now consider a diagonal perturbation of the form:

$$E = \text{diag}\,(e_1, e_2, e_3). \tag{5.22}$$

Then

$$A_e(\omega) = A_0(\omega) + E. \tag{5.23}$$

This would correspond to the realistic situation where there are slight changes in the values of the ground spring k_t, depending for example on how the blades are coupled to the disk in bladed disk assemblies [22]. The major difference between the behavior of (degenerate) cyclic systems and generic systems are the following:

(i) For generic systems, all the eigenvalues and the distance between adjacent pairs increases as the coupling k_c increases. Consequently the probability of mode localization decreases as k_c and hence $\lambda(\omega)_0$ increases. On the other hand, perturbations which split the degenerate eigenvalue of cyclic systems turn them into generic systems with pathologically close eigenvalues [7]. Hence for previously cyclic systems whose eigenvalues bifurcate under perturbations, S_F is very large. Therefore such systems are susceptible to mode localization, independent of the values of the coupling strength k_c. Recall that

$$S_F = \| L \|_2^2,$$

where

$$l_{ij} = (\lambda_j - \lambda_i)^{-1} v_i^* \Delta A u_j,\ i \neq j,\ i,j = 1, 2, \cdots n.$$

(ii) Consequently the only way to avoid large values of S_F in such a situation is if and only if $\| v_i^* \Delta A u_j \| \equiv 0$ or in the neighborhood of zero. Perturbations that induce this condition are precisely those that will not induce radical dynamical changes in mistuned cyclic systems. It was already shown that if $\Delta A = \alpha I$, then $\| v_i^* \Delta A u_j \| \equiv 0$

(iii) Of the remaining possible perturbations those that have $\| v_i^* \Delta A u_j \| = \varepsilon << 1$ will produce minimum dynamical changes. All others for which $\| v_i^* \Delta A u_j \|$ is not small will give susceptibility to mode localization, no matter how strong the interblade coupling.

The following numerical example amplifies the above observations. We consider the case of the so-called 'strong coupling', using the following values: $k_c = 9.5$, $k_t = 1$, $a = 20$, $b = 9.5$, $e_3 = 0$, $e_2 = -0.1$, $e_1 = 0.1$. Clearly, the ratio of mistuning to coupling strength is very small. Now, in order to compute the frequency response curves, we need some damping to obtain finite amplitudes at resonance. Assume hysteretic damping of 0.01 for all cases. Without loss of generality, the response to be computed is the direct receptance, i.e. the response of each node to individual excitation. We turn the ring into a linear chain by putting $b = k_{13} = k_{31} = 0$ in equation (5.21). Then A_0 becomes a tridiagonal banded matrix.

The frequency response of the tuned and mistuned systems of the linear chain are shown in Fig 3. The illustration is windowed around one of the resonant frequencies of the coupled system. Notice that, at the tuned state, the amplitudes of nodes 1 and 3 are equal on account of symmetry, while that of node 2 is double that magnitude.

Because the system is now generic, and therefore exhibits modal stability, all nodes have almost the same response patterns and magnitudes as in the tuned system. This is also the case when we change the sign of e_2, from -0.1 to 0.1.

When we repeat exactly the same procedure for the circulant system, a very different picture is obtained. Fig 4 shows the response of individual nodes compared with the tuned case. This case corresponds to a 2-parameter perturbation, with $e_1 = 0.1$, $e_2 = -0.1$, $e_3 = 0$.

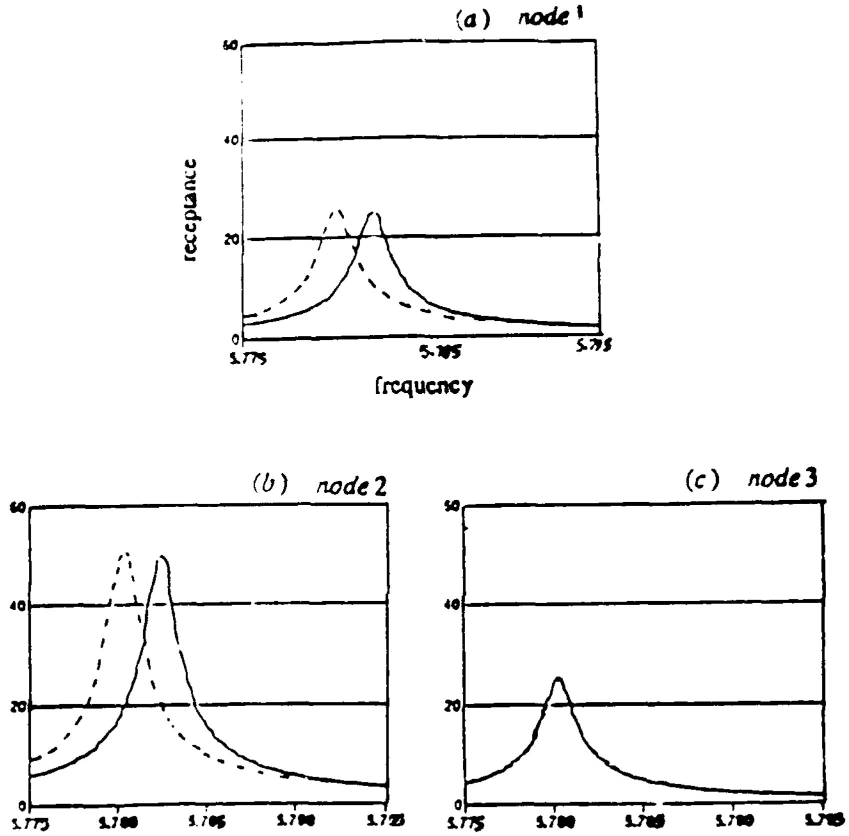

Fig. 3. *Effect of mistuning on the response curves of the linear chain. Note the preservation of the shape of the curves around resonance, and the minimal difference in the peak amplitudes of the tuned and mistuned systems (- - - - tuned systems; _______ mistuned system).*

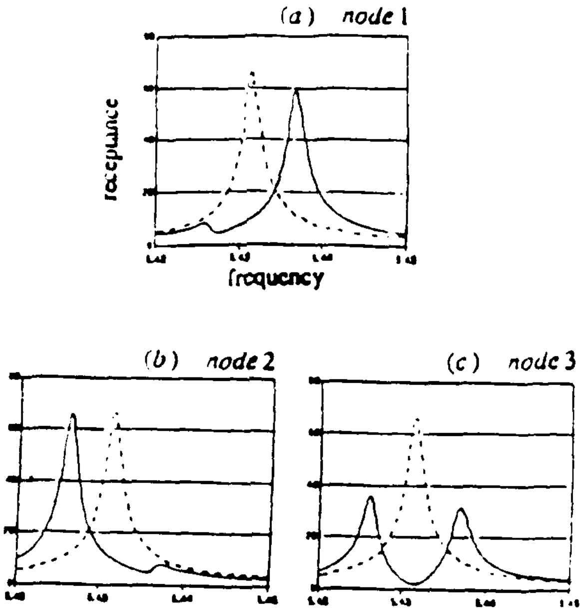

Fig. 4. *Effect of two parameter mistuning on the response curve of the cyclic chain. Note the severe reduction in the amplitude at node 3, which is only 50% of the tuned system (- - - - tuned systems; ______ mistuned system).*

Notice that the node with zero mistuning (mode 3) now has a reduction in amplitude of almost 50%. This extremely unequal amplitude distortion (Fig 4) is the case no matter how small the magnitude of the perturbation is, so long as we keep the *form* of mistuning, and the mistuning does not actually vanish.

If we now change the mistuning matrix in a very small way, by making $e_2=0.1$, we obtain the response curves in Fig 5. We now notice a substantial difference in the geometry of the curves in Fig 5, compared to those in Fig 4. Thus, a very small change in the perturbation matrix, now results in a considerable difference in the vibration response at the individual nodes. The question of which node will be most responding, or the one having the least amplitude, is now not as easy as one would have expected. In Fig 4, it is node 3, while it is node 2 in Fig 5. In fact, the amplitude of node 3 has been increased by about 100% from Fig 4 to Fig 5, merely by changing only one entry in the system matrix from 19.9

to 20.1, a change of less than 1% !

The foregoing examples, based on a simple 3 degrees of freedom model of a circular ring or disk only, illustrates the instability induced by cyclicity. It is clear that the qualitative conclusions to be drawn from Fig 4 are inconsistent with those from Fig 5, although the difference between the two mistuned matrices is very small indeed. We emphasize that these results, obtained for just a cyclic chain, are not necessarily applicable to bladed disks in all generality, especially those models in which cyclicity is ignored. However, when bladed disk systems are well-modeled to include the effects of blade coupling, blade or disk mistuning and *cyclicity*, similar distortions in the geometry of the frequency response curves can result. The subject is currently under investigation by us.

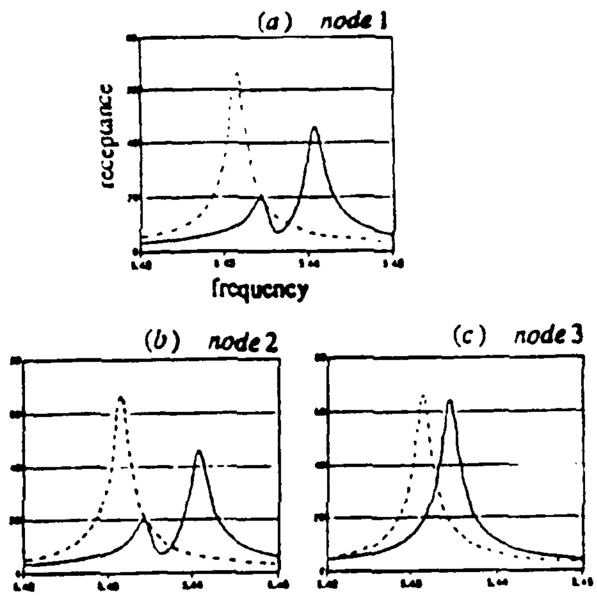

Fig. 5. *Effect of one-parameter mistuning on the response curve of the cyclic chain. Note the symmetrical unfolding of the degenerate singularity (- - - - tuned systems; _______ mistuned system).*

VI. Conclusions

(i) For generic systems, to which linear periodic chains of oscillators belong, differential parameter perturbations are significant for the system dynamics only under weak coupling conditions when the mistuning to coupling ratio exceeds unity (Example 1). Under all other conditions that do not induce eigenvalue degeneracy; small magnitudes of mistuning, or the type of mistuning, is irrelevant to system dynamics.

(ii) For degenerate systems to which a tuned cyclic system with circulant dynamic matrices belongs, it is not just the mistuning to coupling ratio which is significant in the determination of the perturbed system dynamics. The type of mistuning assumes a far greater importance than the mistuning to coupling ratio. All types of mistuning that move the system either across the boundary of the bifurcation set, or from one fiber bundle of the degenerate set to another within Q_v will lead to topological catastrophes [15].

Acknowledgments

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APPENDIX 4

The Stability of Frequency Response Curves

SUMMARY

In this Appendix, we highlight one of the results obtained so far, namely: that the modes of vibration of cyclic structures are unstable under arbitrarily small perturbation. This instability is not the usual ill-conditioned problem of numerical analysis. It has nothing to do with the computational algorithm. The eigenvector instability results because the perfect cyclic system is "physically ill-conditioned", since a very small perturbation changes its dynamics characteristics dramatically. This is significant because many aerospace structures have circular profiles. The implication of eigenvector instability for modal control, forced response amplitudes, sensitivity analysis, etc, therefore needs further investigation.

Eigenvector Stability, Forced Response, and Turbine Blade Failure

The structural integrity of turbine blades used in jet propulsion systems is sometimes compromised by the rare, but very dangerous, failure of some "rogue blades". This problem has been addressed by different investigators of the mistuning problem. However, they often obtained conflicting results.

This is because the response obtained from each rotor studied by each author depends on the eigenvectors of the rotor system matrix A. In general, the matrix A will be different for each model used by each author, although the difference may be very small. In fact, mistuning is usually small.

However, the problem created by mistuning is not always small. Thus, although each A in the family is differentiably dependent on the mistuning parameter ε in the neighborhood of the origin of E, the corresponding eigenvectors is not.

Consequently two almost similar rotors may produce dramatically different vibration responses, if their respective system matrices are different perturbations of the same nominal matrix. An effective demonstration of unstable frequency response curves in a simple 3 degree of freedom cyclic system is given in the following examples. First, we examine the instability of eigenvectors, then the instability of frequency response curves.

Numerical Examples Illustrating Eigenvector Instability

Example 1

At least three coordinates are required to define a cyclic system uniquely. Therefore, we consider the simplest possible example: a 3×3 circulant matrix with real elements, $a,\ b \in \mathbb{R}$.

$$a0 = \begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix}, \tag{A4.1}$$

Using the following perturbation matrices, where $\varepsilon \in \mathbb{R}$ is a very small parameter, we can generate two matrices $A_1 = a0 + E_1$ and $A_2 = a0 + E_2$ that are very close, and such that these depend smoothly on ε, and as $\varepsilon \to 0$, $A_1 \to a0 \leftarrow A_2$. Thus, if

$$E_1 = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad E_2 = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & -\varepsilon & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \text{(A4.2,A4.2)}$$

then

$$A_1 = \begin{bmatrix} a+\varepsilon & b & b \\ b & a+\varepsilon & b \\ b & b & a \end{bmatrix}, \tag{A4.4}$$

and

$$A_2 = \begin{bmatrix} a+\varepsilon & b & b \\ b & a-\varepsilon & b \\ b & b & a \end{bmatrix}, \tag{A4.5}$$

Note that $||A_1|| = ||A_2||$, where $||\cdot||$ is some norm.

Consider, for an illustration, a situation when $a = 100$, $b = 45$, $\varepsilon = 0.1$. We can compute the eigenvalues of A_1 and A_2 respectively as

$$\Lambda_1 = \mathrm{diag}\,(10.0667,\ 145.0333,\ 145.1000) \tag{A4.6}$$

and

$$\Lambda_2 = \mathrm{diag}\,(10.0000,\ 144.9423,\ 145.0578) \tag{A4.7}$$

Notice that the eigenvalues of the two matrices are very close. If we now compute the corresponding eigenvectors, we get

$$U_1 = \begin{bmatrix} .9993 & -.5004 & 1.0000 \\ .9993 & -.5004 & -1.0000 \\ 1.0000 & 1.0000 & 0 \end{bmatrix} \tag{A4.8}$$

$$U_2 = \begin{bmatrix} .9985 & -.2681 & 1.0000 \\ 1.0000 & 1.0000 & -.2678 \\ .9993 & -.7328 & -.7313 \end{bmatrix} \tag{A4.9}$$

We now notice a significant difference between the eigenvectors at modes 2 and 3 of matrices A_1 and A_2 respectively. For example, there is no node (a point where displacement is zero) in the third mode of U_2, whereas there exists such a node in U_1.

Example 2

In the second example, we consider the following circulant; its elements are complex but the matrix is not symmetric. It may be regarded as a deformation of a symmetric circulant.

$$A_1 = \begin{bmatrix} 200 + i(-10) & -95 + i(-5) & -95 + i(5) \\ -95 + i(5) & 200 + i(-10) & -95 + i(-5) \\ -95 + i(-5) & -95 + i(5) & 200 + i(-10) \end{bmatrix} \tag{A4.10}$$

We test for modal stability by computing the eigenvalues

$$\Lambda_1 = \mathrm{diag}\left\{10 + i(-10),\quad 286.3398 + i(-10),\quad 303.6603 + i(-10)\right\} \quad \text{(A4.11)}$$

and the eigenvectors

$$U_1 = \begin{bmatrix} 1 + i(0) & -.5 + i(.866) & -.5 + i(-.866) \\ 1 + i(0) & 1 + i(0) & 1 + i(0) \\ 1 + i(0) & -.5 + i(-.866) & -.5 + i(.866) \end{bmatrix} \quad \text{(A4.12)}$$

Now, we apply a very small perturbation to the matrix A_1 to get:

$$A_2 = \begin{bmatrix} 201 + i(-10) & -95 + i(-5) & -95 + i(5) \\ -95 + i(5) & 199 + i(-10) & -95 + i(-5) \\ -95 + i(-5) & -95 + i(5) & 200 + i(-10) \end{bmatrix} \quad \text{(A4.13)}$$

It is clear that the matrices A_1 and A_2 are 'close', since $||E|| \approx 0$, where

$$E = A_1 - A_2 = \begin{bmatrix} -1 + i(0) & 0 + i(0) & 0 + i(0) \\ 0 + i(0) & 1 + i(0) & 0 + i(0) \\ 0 + i(0) & 0 + i(0) & 0 + i(0) \end{bmatrix} \quad \text{(A4.14)}$$

The computed eigenvalues of A_2 are:

$$\Lambda_2 = \mathrm{diag}\left\{9.9977 + i(-10),\quad 286.3218 + i(-10),\quad 303.6807 + i(-10)\right\} \quad \text{(A4.15)}$$

Now, notice what happens to the third eigenvector of A_1 (eq. (A4.12)), as a very small change is made using E, eq. (A4.13), to transform it to A_2. The eigenvector matrix of A_2 is:

$$U_2 = \begin{bmatrix} 0.993 + i(0.000) & -.474 + i(0.820) & -0.556 + i(-.867) \\ 1.000 + i(0.000) & 1.000 + i(-0.00) & -.444 + i(-.861) \\ 0.997 + i(-0.000) & -.531 + i(0.817) & 1.000 + i(0.000) \end{bmatrix} \quad \text{(A4.16)}$$

Again, it should be noted that a very small change in the matrix A_1 induces a significant qualitative difference in the eigenvector at certain modes of A_1 (eq.

A4.10) compared with the corresponding eigenvectors of A_2, (eq. A4.16).

From Modal Analysis, sometimes known as eigenfunction expansion, we know that the forced response amplitudes are related to eigenvectors. Thus, if the eigenvectors are unstable under arbitrary perturbation, then, the forced response curves will also be unstable under arbitrary perturbation. This is illustrated in Figs A4.1 to A4.2 below.

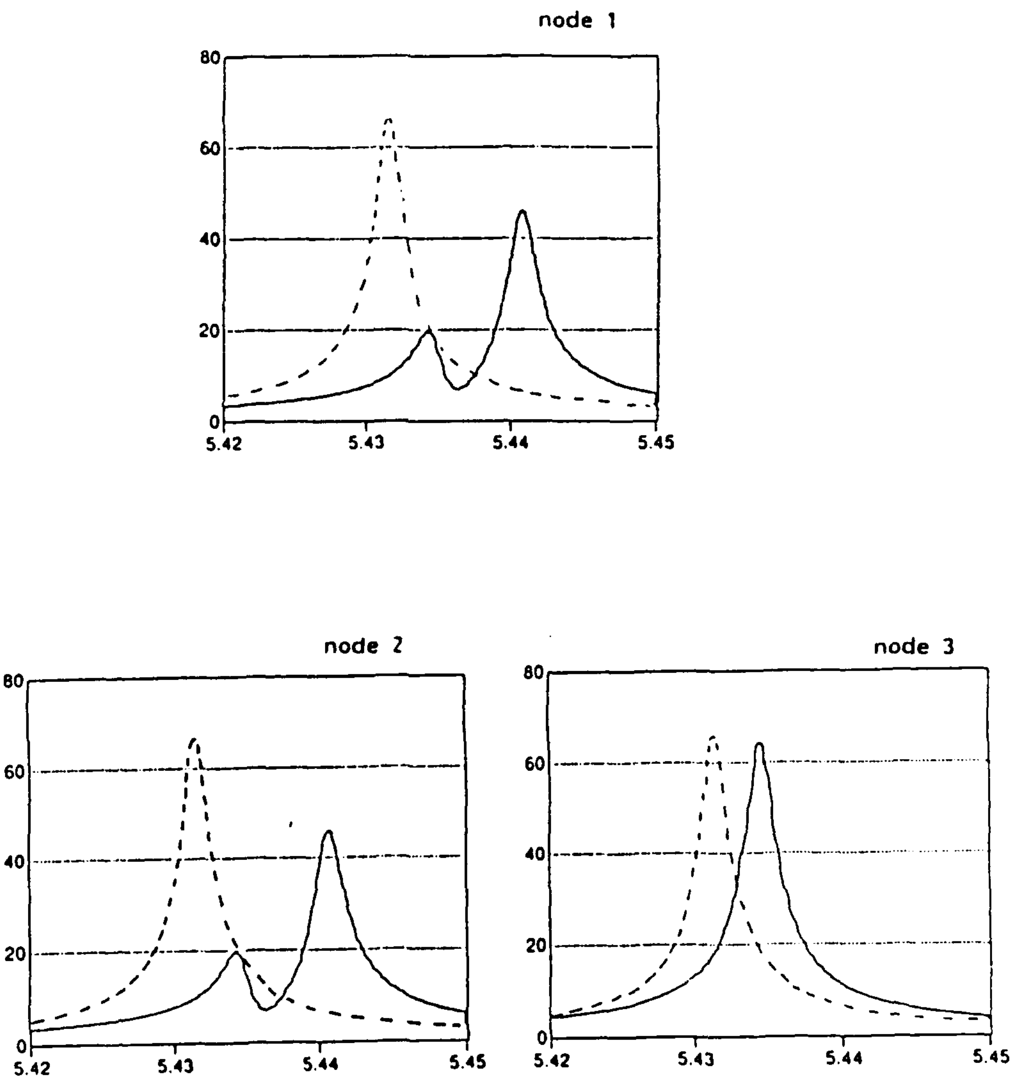
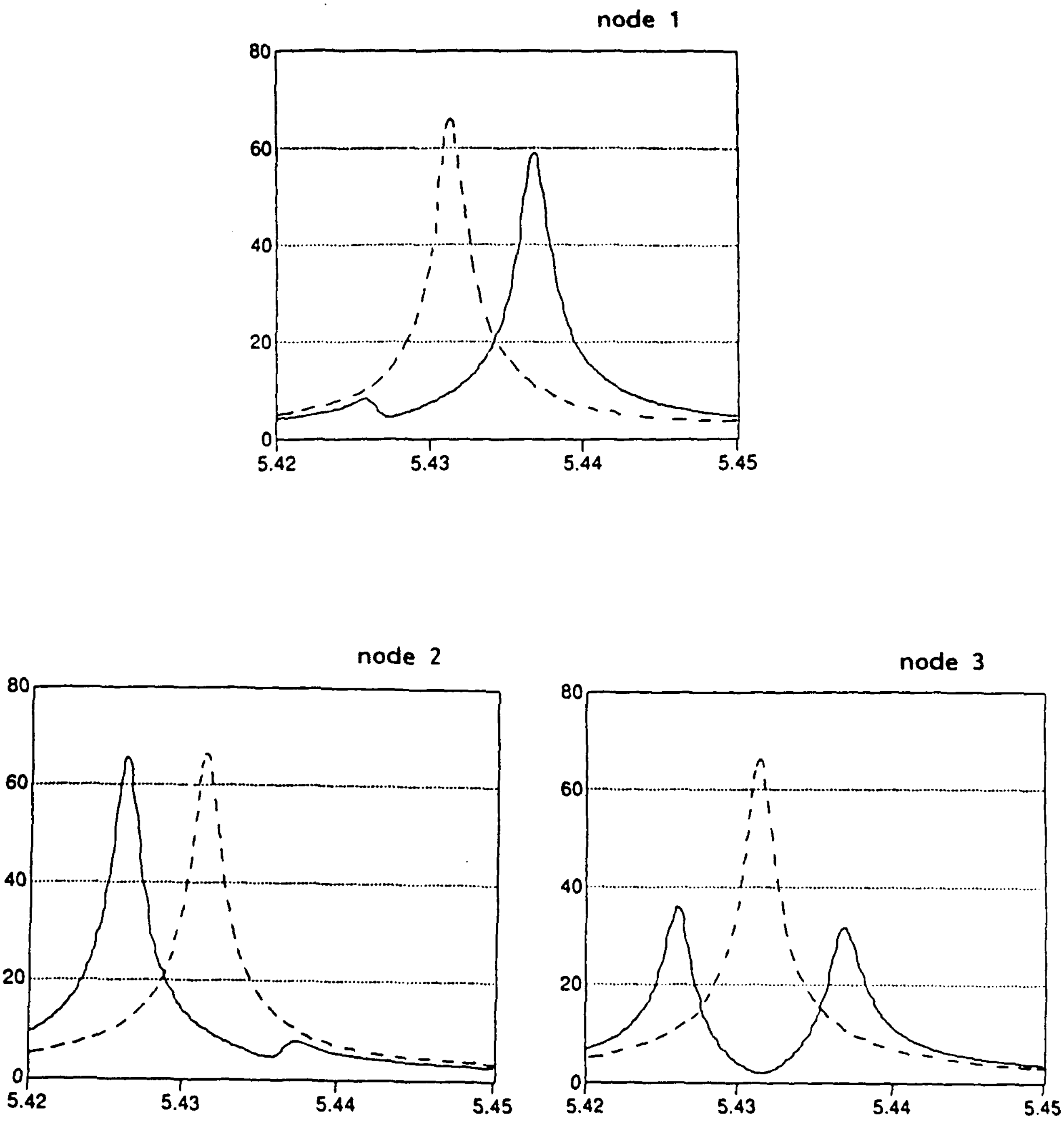


Fig A4.1 Effect of one-parameter mistuning on the response curve of the cyclic membrane. Note the symmetrical unfolding of the degenerate singularity.
(- - - *tuned system;* ______ *mistuned system*)



FigA4.2 Effect of two-parameter mistuning on the response curve of the cyclic membrane. Note the severe reduction in the amplitude at node 3, which is only 50% of the tuned system

(– – – *tuned system;* _____ *mistuned system*)

BIOGRAPHICAL RESUME

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Zentralblat für Mathematik
IEEE Transactions on Automatic Control
IEEE Transactions on Robotics and Automation
IEEE Transactions on Circuits and Systems
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National Science Foundation, Division of Mechanical Engineering and Applied Mechanics (Mechanical Systems).
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Federal Nigeria Graduate Fellowship:	1973 - 1975

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National Center for Interdisciplinary and Policy Studies, University of Nigeria, Founding Member, October 1975 - July 1982.

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1991: Nwokah, O.D.I., Grewal, G.S. (Southern University) Unscheduled Full Envelope Multivariable-QFT Propulsion System Control, WPAFB, $826,000. Project Period: 5/1/91-3/30/95.

Proposals Under Review:

1. Nwokah, O.D.I., High Performance Flexible Mechanical Systems Control by Quantitative Feedback Theory, NSF, $48,000.
2. Nwokah, O.D.I., Bajaj, A.K., Azene, M. (Southern University). Dynamics of Mechanical Systems with Periodic Lattice Structures, $330,000 (3 years). Submitted to ARO.
3. Nordgren, R., Nwokah, O.D.I., Multivariable-QFT Rotorcraft Control, NASA-Ames, $66,000.
4. Gallagher, J., Nwokah, O.D.I., Unscheduled Full Envelope Multivariable Propulsions Systems Control, NASA-LeRC, $66,000.
5. Gallagher, J., Nwokah, O.D.I., Integrated QFT Propulsion-Airframe Control, NASA-Dryden, $66,000.

Proposal In Preparation:

1. Nwokah, O.D.I., Grewal, G.S. (Southern University) Active Vibration Control of Flexible Structures Using Quantitative Feedback Theory. $550,000 (4 years). To be submitted to NASA Ames.

PUBLICATIONS:

A. *Journal Articles:*

1. Nwokah, O.D.I., Stability and the eigenvalues of G(s). *International Journal of Control,* Vol. 22, pp. 125-128, 1975.
2. Araki, M. and Nwokah, O.D.I., Bounds for closed loop transfer functions of multivariable systems, *IEEE Trans. Autom. Control,* Vol. AC-20, pp. 666-670, 1975.
3. Nwokah, O.D.I., Estimates for the inverse of a matrix and bounds for eigenvalues, *Linear Algebra Application*, Vol. 22, pp. 283-292, 1978.
4. Nwokah, O.D.I., The convergence and local minimality of bounds for transfer functions, *International Journal of Control,* Vol. 30, pp. 195-202, 1979.
5. Nwokah, O.D.I., A recurrent issue on the extended Nyquist array, *International Journal of Control,* Vol. 31, pp. 609-614, 1980.
6. Nwokah, O.D.I., A note on the stability of multivariable systems, *International Journal of Control,* Vol. 31, pp. 587-592, 1980.
7. Nwokah, O.D.I., The reduction of transfer function matrices to generalized diagonally dominant form. *Trans. Int. Meas. Control,* Vol. 2, pp. 132-136, 1980.
8. Nwokah, O.D.I., Generalized Nyquist stability criterion for composite feedback control systems, *Large Scale Systems*, Vol. 2, pp. 185-190, 1981.
9. Nwokah, O.D.I., On the stability of multivariable systems, *Systems and Control Letters,* Vol. 2, No. 6, pp. 363-368, 1982.
10. Nwokah, O.D.I., The stability of linear multivariable systems, *International Journal of Control,* Vol. 37, pp. 623-629, 1983.
11. Nwokah, O.D.I., Synthesis of feedback systems for specified time domain insensitivity to interaction induced plant ignorance, *International Journal of Control,* Vol. 37, pp. 421-428, 1983.
12. Nwokah, O.D.I., Multiple gain parameter multivariable root locus, *Systems and Control Letters,* Vol. 3, pp. 197-201, 1983.
13. Nwokah, O.D.I., Composite matrix inverses and generalized Gershgorin sets, *Math. Proc. Camb. Phil. Soc.* V95, pp. 267-276, 1984.
14. Nwokah, O.D.I., Synthesis of Controllers for Uncertain Multivariable Plants, *International Journal of Control,* Vol. 40, No. 6, pp. 1189-1206, 1984.
15. Nwokah, O.D.I., On non-singular value based design of controllers for robust stability, *IEE Proceedings*, Vol. 133, Pt. D, pp. 57-64, 1986.
16. Nwokah, O.D.I., The robust decentralized stabilization of complex feedback systems, *IEE Proceedings*, Vol. 134, Pt.D., pp. 43-47, 1987.

17. Nwokah, O.D.I., The quantitative design of robust multivariable feedback systems, *IEE Proceedings*, Vol. 135, Pt.D., pp. 57-66, 1988.

18. Perez, R.A., Nwokah, O.D.I., Integrated propulsion-airframe systems control, *International Journal of Control* (to appear).

19. Nwokah, O.D.I., Thompson, D.F., Algebraic and topological aspects of quantitative feedback theory, *International Journal of Control* 50, 1057-1069, 1989.

20. Le, D.K., Nwokah, O.D.I., Frazho, A.E., Multivariable Decentralized Integral Controllability, *International Journal of Control* (to appear).

21. Nwokah, O.D.I., M-matrix methods in quantitative feedback theory, *Internal Journal of Control* (to appear).

22. Nwokah, O.D.I., Strong robustness in uncertain multivariable systems, in P. Borne et al: *Computing and Computers for Control Systems*, 401-408, J. C. Batzer AG. Scientific Publishers, 1989.

23. Perez, R.A., Nwokah, O.D.I., Integrated propulsion-airframe system control, *International Journal of Dynamics and Control* (to appear).

24. Perez, R.A., Nwokah, O.D.I., Multivariable control of an integrated propulsion-airframe system, Lecture Notes in Control and Information Science, No. 398, 371-380, Springer-Verlag, 1990.

25. Nwokah, O.D.I., Afolabi, D., Damra F., On the modal stability of imperfect cyclic systems, controls and dynamic systems, 35, 137-164, 1990 (Academic Press, Ed., C.T. Leondes).

26. Nwokah, O.D.I., Perez, R.A., On multivariable stability in the gain space, *Automatica* (To Appear).

27. Perez, R.A., Nwokah, O.D.I., Shahin, R.A., Control structure assignment in decentralized stabilization, *ASME Journal of Dynamic Systems and Control* (To Appear).

28. Happawana, G.S., Bajaj, A.K., Nwokah, O.D.I., A singular perturbation perspective on mode localization, *Journal of Sound and Vibration* (To Appear).

29. Thompson, D.F., Nwokah, O.D.I., Frequency response specifications and sensitivity functions in quantitative feedback theory, *Automatica* (To Appear).

30. Happawana, G.S., Bajaj, A.K., Nwokah, O.D.I., Mode localization in linear chain and cyclic periodic systems, *J. Sound and Vibrations* (To Appear).

B. *Book Chapters*

1. Nwokah, O.D.I., Critical mass and University research in a developing environment. In Onwuka, R.I. and Ghista, D.: African Development: OAU/ECA Lagos plan of action and beyond, Brunswick Publishers, Lawrenceville, Virginia, 1985.

2. Nwokah, O.D.I., The problems of Engineering academics in a developing environment: The Nigerian experience, In Ukaegbu, C.C. and Iroegbu, C.U. (Eds.): Toward a manpower-oriented development policy for Nigeria, Nok Publishers, New York and Lagos, (1987).

3. Skinner, E.C. and Nwokah, O.D.I., Communication and continuity in the diaspora: Some personal reflections on cultural connections, In Gay, G., and Baber W.L., (Eds.): Expressively Black; The cultural basis of ethnic identity, pp. 321-344, Praeger, New York, 1987.

4. O.D.I. Nwokah, Neoclassical Multivariable Feedback Control. (Research Monograph). Currently under revision.

C. *Refereed Conference Proceedings:*

1. Nwokah, O.D.I., The construction of Lyapunov functions for composite dynamical systems. Proceedings of the 1st Nigerian Congress of theoretical and applied mechanics, (NICONTAM '76) pp. C254-C267, 1976.

2. Nwokah, O.D.I., The design of linear multivariable systems (Invited Paper), Proc. IEEE Mexicon 1976, Conference. Mexico City 1976, Paper 1.9.

3. Nwokah, O.D.I., Progress in the design of multivariable control systems, Proc. 4th IFAC Symposium on Computer aided design of multivariable technological system, Fredericton, Canada, 1977, Paper 3.1.

4. Nwokah, O.D.I., Hadamard transfer matrices and diagonal dominance. Proc. IEEE Mexicon 1977 Conference. Mexico City 1977, Paper 4.7.

5. Nwokah, O.D.I., Synthesis of feedback systems for specified time domain insensitivity to interaction induced plant ignorance. Proc. 6th IFAC Symposium on computer aided design of multivariable technological systems, Purdue, Indiana, U.S.A. 1982.

6. Nwokah, O.D.I., The robust decentralized stabilization of composite feedback systems, Proc. 4th IFAC Symposium on Large Scale Systems, Zurich, Switzerland, 1986.

7. Nwokah, O.D.I., On decentralized control of complex feedback systems, Proceedings American Control Conference, Seattle, WA, Vol. 1, pp. 326-331, 1986.

8. Nwokah, O.D.I., The quantitative design of robust multivariable control systems, Proc. 25th IEEE Conference on Decision and Control, Athens, Greece, pp. 16-24, 1986.

9. Nwokah, O.D.I., Pseudo-derivative feedback control, Proceedings American Control Conference, Minneapolis, MN, pp. 1811-1814, 1987.

10. Nwokah, O.D.I., Strong Robustness in Uncertain Multivariable Systems, Proc. 27th IEEE Conference on Decision and Control, Austin, Texas, December 1988.

11. Nwokah, O.D.I., Strong diagonal dominance and closed loop performance, Proc. ASME Winter Annual Meeting, Chicago, Illinois, December 1988.

12. Nwokah, O.D.I., Strong robustness in uncertain multivariable systems, Proc. 12th IMACS World Congress on Scientific Computation, Paris, 1988.

13. Nwokah, O.D.I., Thompson, D.F., Algebraic and topological aspects of quantitative feedback theory, Proc. American Control Conferences, Pittsburgh, PA, June 21-23, 1989.

14. Afolabi, D., Nwokah, O.D.I., Perturbations of periodic systems, Proc. 21st Midwest Mechanics Conference, Michigan Tech. University, July 1989.

15. Afolabi, D., Nwokah, O.D.I., The frequency response of mistuned cyclic systems, Proc. ASME Vibration Conference, Montreal, PQ, Canada, September 17-19, 1989.

16. Burhoe, J.C.A., Nwokah, O.D.I., Multivariable control of a biaxial machine tool, ASME Winter Annual Meeting, San Francisco, December 1989.

17. Perez, R.A., Nwokah, O.D.I., The integrated control of a propulsion-air frame system, ASME Winter Annual Meeting, San Francisco, December 1989.

18. Thompson, D.F., Nwokah, O.D.I., Stability and optimal design in quantitative feedback theory, ASME Winter Annual Meeting, San Francisco, December 1989.

19. Nwokah, O.D.I., Thompson, D.F., Robust low order controller design in quantitative feedback theory, Proc. 28 CDC, Tampa, FL, December 13-15, 1989.

20. Perez, R. A., Nwokah, O.D.I., Multivariable control of an integrated propulsion-airframe system, 3rd USC Workshop on Control Mechanics, January 1990.

21. Happawana, G.S., Nwokah, O.D.I., Bajaj, A.K., Afolabi, D., On the dynamics of perturbed cyclic symmetric systems, 3rd USC Workshop on Control Mechanics, January 1990.

22. Thompson, D.F., Nwokah, O.D.I., Frequency response specifications and sensitivity functions in quantitative feedback theory, Proc. ACC, San Diego, 599-604, 1990.

23. Thompson, D.F., Nwokah, O.D.I., Optimal loop synthesis in quantitative feedback theory, Proc. ACC, San Diego, 626-631, 1990.

24. Afolabi, D., Nwokah, O.D.I. Effects of mild perturbations on the dynamics of structures with circulant matrices, Proc. 1990 AIAA/ASME/ASCE/AHS, Structures, Structural Dynamics and Materials Conference, Long Beach, CA, 1990.

25. Happawana, G.S., Afolabi, D., Nwokah, O.D.I., Bajaj, A.K., On the dynamics of perturbed symmetric systems, Proc. 11th U.S. National Congress of Applied Mechanics, Tucson, AZ, May 21-25, 1990.

26. Nwokah, O.D.I., Perez, R.A., On multivariable stability in the gain spaced, Proc. IEEE Conference on Decision and Control, Honolulu, Hawaii, December 1990.

27. Nwokah, O.D.I., Thompson, D.F., Perez, R.A., On the existence of QFT controllers, ASME-WAM, Dallas, TX, 1990.

28. Nwokah, O.D.I., Jayasuriya, S., Chait Y., Constrained robust disturbance accommodation by quantitative feedback theory (QFT), 4th USC Workshop on Control Mechanics, University of Southern California, January 22-24, 1991.

D. *Technical Reports:*

1. Nwokah, O.D.I., On design of multivariable control systems, Control Systems Centre report No. 429, UMIST, Manchester, England, September 1978.
2. Nwokah, O.D.I., Weak diagonal dominance and the Nyquist array, Research report IAS (1980), Manchester Polytechnic, Manchester, England.
3. Nwokah, O.D.I., Strongly restricted interaction and minimum variance controllers for multivariable systems. Research report IAS, (1980) Manchester Polytechnic, Manchester, England.
4. Nwokah, O.D.I., The assignment of loop interaction in linear multivariable systems. Research report IAS (1980), Manchester, England.
5. Nwokah, O.D.I., Multivariable describing functions and the Nyquist array, Research report IAS (1979), Manchester Polytechnic, Manchester, England.
6. Nwokah, O.D.I., The integrity and robustness of systems with integral control, Control Systems Technical Note #30, Dept. of Electrical Engr., University of Notre Dame, Oct. 1984.
7. Nwokah, O.D.I., The robust decentralized stabilization of composite dynamical systems, Control Systems Technical Note #31, Dept. of Electrical Engr., University of Notre Dame, Oct. 1984.
8. Nwokah, O.D.I., The robust multi-level hierarchical control of composite feedback systems, Control Systems Technical Note #32, Dept. of Electrical Engr., University of Notre Dame, Oct. 1984.
9. Nwokah, O.D.I., On non-singular value based design of controllers for robust stability, Control Systems, Technical Note #33, Dept. of Electrical Engr., University of Notre Dame, Dec. 1984.
10. Nwokah, O.D.I., Sensitivity and robustness optimization of linear multivariable systems, Control Systems Technical Note #35, Dept. of Electrical Engr., University of Notre Dame, March 1985.

E. *Some Invited Presentations.*

1. The Design of Linear Multivariable Systems - University of London (Imperial College), Dept. of Electrical Engineering, June 18, 1975.
2. Rapid Changes of State in the Nigerian Socio-economic System: Applications of Singularity Theory, University Engineering Lecture Series, University of Nigeria, March 1976.
3. Progress in the Design of Multivariable Systems, Control Systems Center, University of Manchester Institute of Science and Technology, July 1977.
4. Applications of Catastrophe Theory to Modelling of the Nigerian Socio-economic System. Nigerian Institute for Social and Economic Research, University of Ibadan, April 1977.
5. Interaction Indices for Multivariable Systems, Joint Control Seminar Series, Sheffield University/Sheffield Polytechnic, Dept. of Electrical Engineering, Sheffield, England, November 1979.

6. Multivariable Control of Aircraft Fatigue Testing Rigs, Department of Electrical and Electronic Engineering, University of Surrey, England, March 1980.

7. A Control Theory Component to the German Academic Exchange Program (DAAD) on Industrial Mathematics, Department of Mathematics, University of Ibadan, Nigeria, February 1981.

8. Frequency response methods for the control of uncertainty in multivariable plants, Case Western Reserve University, Department of Electrical Engineering and Applied Physics, Cleveland, Ohio, April 1985.

9. Polynomial based feedback design and Bode's frequency response conditions, Department of Electrical Engineering, Seattle University, Seattle, Washington, March 1985.

10. Are Engineering Control and H^∞ Optimization Compatible, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, February 1985.

11. M-matrices and Quantitative Robustness Measures in Multivariable Uncertain Systems, Harris Corporation, Government Aerospace Systems Division, Melbourne, Florida, August 1987.

12. Practical Aspects of Sensitivity Reduction in Engineering Control Systems, Purdue University at Indianapolis, School of Engineering and Technology, November 1987.

13. The Dynamics of Mistuned Cyclic Systems, Electric Power Research Institute, Palo Alto, CA, October 13, 1988.

14. Integrated Dynamics and Control of Propulsion-Airframe System, Duke University, Durham, NC, Dept. of Mechanical Engineering and Materials Science, April 5, 1989.

15. Application of QFT to Engine-Airframe Integration, University of Illnois at Urbana-Champaign, Dept. of Mechanical and Industrial Engineering, April 18, 1989.

16. Model Reference QFT Application to Ship Propulsion Systems, Texas A&M Univesity, Dept. of Mechanical Engineering, March 22, 1991.

F. *Thesis Supervision.* (Major Advisor)

(i) Ph.D.

David Thompson:	Optimal Loop Shapes in Quantitative Feedback Theory; August 1990.
Ronald Perez:	Integrated Propulsion-Airframe Dynamics and Control; August 1990.
Germu Happawana:	Dynamics and Control of Vibration in Bladed Disk Assemblies (Degree Expected December 1992)
Sobita Samanarayake:	The Algebraic and Topological Dynamics of Periodic Systems (Degree Expected December 1993)

Chin-Horng Yau:	Generic Modelling and Control of Integrated Aero Engine/Air Frame Systems (Degree Expected December 1992)
Richard Nordgren	Robust Multivariable-QFT Rotorcract Control (Degree Expected December 1993)

(ii) MSME (Thesis)

Alley Butler:	Analysis of the Control of Kamyr Digester Based on the Purdue Kamyr Digester Model; August 1988.
John Burhoe:	Coordinated Control of Contour and Tracking Accuracy in X-Y Tables; December 1988.
Ali Shahin:	Modeling and Control of the PD 514-1 STOVL Engine (December 1990)
Joseph E. Gallagher	Multivariable - QFT Control of the Allison PD 514-1 STOVL Engine (December 1991)

(iii) MSME (Projects)

Robert Matzen:	Experimental Modelling and Analysis of Contour Tracking X-Y Tables; May 1988.
Nader Motamedi:	Vibration Dynamics of Bladed Disk Assemblies; May 1988.
Christine Treml:	Computational Issues in Signal Processing (Supervised by P. Sherman), May 1988.
Richard Walker:	Controller Design for Coordinated Control Applications; December 1988.
Douglas Starfield:	Expert Systems Control Software Development," August 1989.
Waylon Wang	Active Control of Mechanical Impedance December 1989.

(iv) MSME (By Course Work Only)

Richard Strehler:	May 1987.

(v) MSEE (Thesis)

Gloria Nnebedum:	Modelling and Control of the Fermentation Process in Garri Production, University of Nigeria, December 1983.
Alex Anyaegbunam:	Microprocessor Control of the Garri Frying Machine, University of Nigeria, July 1984.
Clair Courtine:	Nyquist Array Based Methods for the Control of Multivariable Systems, Purdue University (Co-advisor: J. Chiasson), May 1988.

G. *Teaching Experience: September 1975 to Present.*

Courses Taught:
(i) Undergraduate: Control Engineering, Engineering Laboratories, Engineering Mathematics, Modelling of Mechanical and Electromechanical Systems, Electronic Circuits, Digital Logic Design, Electronic Circuit Design,

(ii) Graduate: Linear System Theory, Theory and Design of Control Systems, Multivariable Control System Design.

(i) University of Nigeria

ECE 512:	Control Engineering, 1975, 1976, 1977, 1978, 1979. (Core Undergraduate)
Engr. 301:	Engineering Mathematics 1, 1975 -- 1984. (Core Undergraduate)
Engr. 401:	Engineering Mathematics 2, 1975 -- 1984. (Core Undergraduate)
ECE 412:	Instrumentation, 1980 -- 1984. (Core Undergraduate)
EE 312:	Modelling of Mechanical and Electromechanical Systems. (Core Undergraduate)
ECE 612:	Linear System Theory. (Core Graduate)

(ii) Manchester Polytechnic

EE 214:	Electronics, Fall 1979, Spring 1980. (Core Undergraduate)
EE 314:	Automatic Control, Fall 1979, Spring 1980. (Core Undergraduate)

(iii) University of Notre Dame

EE 211:	Digital Logic Design, Fall 1984, Spring 1985. (Core Undergraduate)
EE 311:	Electronic Circuits Design, Spring 1985. (Core Undergraduate)

(iv) Purdue University

ME 375:	Modelling and Analysis of Physical Systems, Spring 1987 Fall 1988. (Core Undergraduate)
ME 463:	Mechanical Engineering Design, Fall, 1988, Spring 1989 (Core Undergraduate)
ME 475:	Automatic Control, Fall 1985, Spring 1986, Fall 1986.
ME 475L:	Mechanical Engineering Control Lab, Fall 1987, Spring, 1988, Spring 1989 (Core Undergraduate)
ME 575:	Theory and Design of Control Systems, Fall 1987. (Core Undergraduate)
ME 597N:	Multivariable Control Systems Design, Fall 1986. (Graduate)
ME 697N:	Multivariable Control Systems Design, Spring 1988. (Graduate)

H. *New Courses Developed*

Reorganized and updated ME 475 laboratory experiments with a grant from AT&T Foundation.

Contributed to the reorganization of the ME 475 course structure and material.

Introduced and developed the new graduate course: ME 697N.
Multivariable Control System Design
This is the only frequency domain control offering at Purdue

I. *Works under Review*

1. Perez, R.A., Nwokah, O.D.I., Thompson, D.F., Almost decoupling by quantitative feedback theory, Submitted to ASME Journal Dynamic Systems and Control.

2. Nwokah, O.D.I., Jayasuriya, S., Chait, Y, Parametric robust control by quantitative feedback theory, Submitted to AIAA Journal Guidance and Control.

3. Jayasuriya, S., Nwokah, O.D.I., Yaniv, O., The Benchmark problem solution by quantitative feedback theory. (Invited 1991 ACC Paper) Also submitted for a special issue of AIAA Journal Guidance and Control.

4. Happawana, G.S., Bajaj, A.K., Nwokah, O.D.I., A singular perturbation analysis of eigenvalue veering and mode localization in perturbed linear chain and cyclic systems. Submitted to Journal of Sound and Vibrations

J. *Professional Activities*

1. Organizer and Chairman: Session on Quantitative Feedback Theory, ASME-WAM, San Francisco, December 1989.

2. Organizer and Co-chair: Session on QFT Approach to Robust Control, ACC, San Diego, CA, May 1990.

3. Organizer and Chair: Two Sessions on Recent Developments in QFT; ASME-WAM, Dallas, TX, December 1990.

4. Chairman: Session on Linear Stability, IEEE-CDC, Honolulu, Hawaii, December 1990.

5. Organizer and Co-chair: Session Parametric Robust Control by Quantitative Feedback Theory, ACC, Boston, MA, June 1991.

6. Organizer and Chair: 2 Sessions on QFT, ASME-WAM, Atlanta, GA, December 1991.

BIOGRAPHICAL RESUME

ANIL KUMAR BAJAJ

(updated January 91)

Personal Data:

Address:	School of Mechanical Engineering Purdue University West Lafayette, IN 47907
Office Phone:	(317) 494-6896 Home Phone: (317) 463-5487

Birthdate:	April 25, 1951, Allahabad, India

Education:

1976-1981	Ph.D. in Mechanics, University of Minnesota, Minneapolis, Minnesota. (Advisor: Professor P. R. Sethna)
1974-1976	M.Tech. in Mechanical Engineering, Indian Institute of Technology, Kanpur, India.
1968-1973	B.Tech. in Mechanical Engineering, Indian Institute of Technology, Kharagpur, India.

Work Experience:

Aug. 1985-present	Associate Professor, School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907
June-July 1988	Visiting Associate Professor, Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, MN 55455
Feb.-April 1988	Visiting Faculty, Department of Mechanical Engineering Indian Institute of Technology, Kanpur, India

Jan. 1981-July 1985	Assistant Professor, School of Mechanical Engineering, Purdue University, West Lafayette, IN 47907
June-August 1981	Visiting Research Associate, Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, MN 55455
Sept. 1976-Dec. 1980	Teaching and Research Assistant, Department of Aerospace Engineering and Mechanics, University of Minnesota, Minneapolis, MN 55455
Aug. 1974-June 1976	Research Assistant, Department of Mechanical Engineering, Indian Institute of Technology, Kanpur, India

Honors, Activities, Organizations:

American Academy of Mechanics - Member.
ASME - Associate Member
Sigma Gamma Tau (Aero. Eng. Honorary Society) - Member
B.Tech - Institute Silver Medal
Sigma Xi - Member
Society of Engineering Science - Member
Society of Industrial and Applied Mathematics (SIAM) - Member

Society Activities:

Member, ASME Technical Committee on Dynamics of Structures and Systems, Applied Mechanics Division. 1989-present

Recent Teaching Experience:

Purdue University:

ME 270 - Basic Mechanics I (Statics) - S81,F82,F83,F84,F85,SS86,F86,F87,F88,F89,F90
ME 270I - Basic Mechanics I (Independent) - S82
ME 274 - Basic Mechanics II (Dynamics) - F81,SS84,S86,SS87,S89
ME 562 - Advanced Dynamics - S82,S83,S84,S86,S89,S90 (TV),S91
ME 563 - Mechanical Vibrations - F83,S85,SS86,S87,F87
ME 580 - Nonlinear Systems - F81,F82,F84,F85,F86,F88,F89,F90
ME 597T - Analysis and Design of Manipulators - S83,S84,S85
ME 665 - Dynamic Stability of Elastic Systems - S83
ME 697B - Introduction to Bifurcations and Chaos - S87,S90

Other Teaching Related Activities:

1. Revised graduate courses ME 562, ME 580.
2. Completely revised and reorganized ME 665.
3. Introduced new course: Introduction to Bifurcations and Chaos.

University and Departmental Activities

1. Graduate School Fellowship Committee: 1983, 1984, 1985, 1986, 1987.
2. Mechanical Engineering Graduate Committee: 81-82, 82-83, 83-84, 84-85, 85-86.
3. Mechanical Engineering Curriculum Committee: 86-87, 87-88.
4. Dean's Ad Hoc Committee on Undergraduate Students distribution in the Schools of Engineering: 87.
5. Schools of Engineering Academic Personnel Grievance Committee: 1988-89, 1989-1990.
6. Departmental Task Force on Secretarial Services: 1989 Summer.

Refereeing Activities:

ASME Journal of Applied Mechanics
SIAM Journal on Applied Mathematics
ASME Journal of Dynamic Systems, Measurements and Control
ASME Journal of Mechanisms, Transmissions, and Automation in Design
Journal of Sound and Vibration
Dynamics and Stability of Systems
IEEE Transactions on Circuits and Systems
Journal of Fluids and Structures

International Journal of Nonlinear Mechanics
Journal of Fluid Mechanics
ASME Journal of Vibration, Acoustics, Stress, and Reliability in Design
Physica D.
Nonlinear Dynamics

Reviewing Activities:

National Science Foundation, Division of Mechanics, Structures and Materials.
Army Research Office, Division of Engineering Sciences.

Consulting:

Consultant to Rubbermaid Incorporated, Reynolds, IN, in structural re-design of a molded, polyester resin part (1989).

Consultant to Arrowhead Plastic Engineering, Inc., Muncie, IN, in Failure Diagnosis and Redesign of a Plastic Fertilizer Storage Box (1989).

Consultant to O'Reilly, Cunningham, Norton & Mancini, Law Offices, Wheaton, IL, on behalf of LMP Steel, Defendant, in a lawsuit involving stacking of bundles of steel bars constrained by crimped spring steel bands (1989 - present).

Research and Other Grants:

1. XL Summer Grant - Purdue University - 1982 - $3,300.
2. Award from National Science Foundation, Division of Mechanical Engineering and Applied Mechanics for the period September 1982 - February 1985 for project entitled, "Bifurcations in Rotationally Symmetric and Perturbed Mechanical Systems" - $49,991.
3. XL Summer Grant - Purdue University - 1983 - $3,500.
4. a. David Ross Grant - for the project "Dynamic Analysis of Robotic Manipulators With Parametric and External Excitation", - 1984 (with C. M. Krousgrill) - $6,600.

   b. Renewal of above, 1985 - $6,600.
5. PRF International Travel Grant - Purdue University - 1984 - $800.
6. Travel Grant-in-aid-U.S.N.C. on Theoretical and Applied Mechanics, National Research Council - 1984 - $700.
7. AMP Inc. "Study of Parametric Instability of High Speed Elastic Mechanisms", September 1984 - August 1985 (with A. Midha) - $12,000.
8. a. David Ross Grant - for the project "Qualitative Studies in the Nonlinear Dynamics of Compliant Offshore Structures" - 1985 (with C. M. Krousgrill) - $6,600.

   b. Renewal of above, 1986 - $6,600.
9. Govt. of Indonesia grant for the project "M.S. - Mechanical Engineering - Andi Mahyuddin" August 1986 - July 1988 (with Mr. A. Mahyuddin) - $13,458.
10. Travel Grant - U. S. National Committee on Theoretical and Applied Mechanics, National Research Council - 1988 - $1,025.
11. Award from AFOSR, Structural Mechanics Program, Bolling AFB, Washington, D.C. for November 1988 - October 1990 for project "Vibration Dynamics and Control of Bladed Disk Assemblies" - $124,049 (Co-PI with Professor O. D. Nwokah).
12. a. David Ross Grant - for the project "Analytical and Experimental Investigations of Chaotic Response in Structures with Nonlinear Modal Interactions" - 1989 (with C. M. Krousgrill) - $7,890.

    b. Renewal of above, 1990 - $9,000.
13. Dean's Club "Creative Undergraduate Instructional Project" grant for the project - Development of Graphical Display and Simulation Software for the Enhancement of Visualization in Undergraduate Mechanics Education - 1990 (with C. M. Krousgrill and D. C. Anderson) $9,000.
14. Hewlett - Packard Equipment Grant (through the Head of the School) for instructional laboratory in dynamics of mechanical systems - 1990 (with C. M. Krousgrill and A. Midha) $39,735.

15. Award from Engineering Sciences Division, Army Research Office, Dept. of the Army for the period November 1, 1990 - October 31, 1993 for project "Modal Interactions and Complex Responses in Weakly Nonlinear Multi-Degree-of-Freedom Mechanical Systems" - $290,000 (PI with Prof. Davies as Co-PI).

M.S. Theses Directed:

1. S. Tousi, School of Mechanical Engineering: Period-Doubling Bifurcations and Modulated Motions in Forced Mechanical Systems (1984).

2. A. I. Mahyuddin, School of Mechanical Engineering: A Comparative Parametric Stability Study for Flexible Cam-Follower Systems due to Varying Cam Profiles (Dec. 1988) (Co-Advisor with Prof. A. Midha).

M.S. Students Advised (Non-thesis Option)

1. Thomas Berglund, Tod Dalrymple, Carol Dittrich, David Ehinger, Leslie Grundman, Gerald Merrill, Scott Pantaleo, Timothy Rorick, David Smith, David W. Turner

Ph.D. Theses Directed:

1. M. Ilkhani-Pour, School of Mechanical Engineering: Effect of a Viscoelastic Boundary Inset on Vibrational Characteristics of a Beam (Dec. 1984) (Co-Advisor with Prof. E. C. Ting, CE).

2. D. A. Streit, School of Mechanical Engineering: Dynamics and Stability of Robotic Manipulators with Parametric Excitation (Aug. 86) (Co-Advisor with Prof. C. M. Krousgrill).

3. Y. M. Huang, School of Mechanical Engineering: Nonlinear Dynamics of Low Order Models of Offshore Structures (Dec. 87) (Co-Advisor with Prof. C. M. Krousgrill).

4. S. Tousi, School of Mechanical Engineering: Directional Stability of Road Vehicles (Dec. 88) (Co-Advisor with Prof. W. Soedel).

5. J. M. Johnson, School of Mechanical Engineering: On the Vibration of Stretched Strings (May 1989).

Journal Publications:

1. Bajaj, A. K. and Garg, V. K., "Linear Stability of Jet Flows," J. Appl. Mech., Vol. 44, Sept. 1977, pp. 378-384.
2. Sethna, P. R. and Bajaj, A. K., "Bifurcations in Dynamical Systems with Internal Resonance," J. Appl. Mech., Vol. 45, Dec. 1978, pp. 895-902.
3. Lundgren, T. S., Sethna, P. R., and Bajaj, A. K., "Stability Boundaries for Flow Induced Motions of Tubes with an Inclined Terminal Nozzle," J. Sound and Vib., Vol. 64, 1979, pp. 553-571.
4. Bajaj, A. K., Sethna, P. R., and Lundgren, T. S., "Hopf Bifurcation Phenomena in Tubes Carrying a Fluid," SIAM J. Appl. Math., Vol. 39, 1980, pp. 213-230.
5. Bajaj, A. K., "Bifurcating Periodic Solutions in Rotationally Symmetric Systems," SIAM J. Appl. Math., Vol. 42, Oct. 1982, pp. 1078-1098.
6. Bajaj, A. K. and Sethna, P. R., "Bifurcations in Three-Dimensional Motions of Articulated Tubes, Part 1: Linear Systems and Symmetry," J. Appl. Mech., Vol. 49, Sept. 1982, pp. 606-611.
7. Bajaj, A. K. and Sethna, P. R., "Bifurcations in Three-Dimensional Motions of Articulated Tubes, Part 2: Nonlinear Analysis," J. Appl. Mech., Vol. 49, Sept. 1982, pp. 612-618.
8. Bajaj, A. K. and Sethna, P. R., "Flow-Induced Bifurcations to Three-Dimensional Oscillatory Motions in Continuous Tubes," SIAM J. Appl. Math, Vol. 44, Apr. 1984, pp. 270-286.
9. Bajaj, A. K., "Interactions Between Self and Parametrically Excited Motions in Articulated Tubes," J. Appl. Mech., Vol. 51, June 1984, pp. 423-429.
10. Sethna, P. R., Meyer, K. R., and Bajaj, A. K., "On the Method of Averaging, Integral Manifolds and Systems with Symmetry," SIAM J. Appl. Math., Vol. 45, June 1985, pp. 343-359.
11. Tousi, S. and Bajaj, A. K., "Period-Doubling Bifurcations and Modulated Motions in Forced Mechanical Systems," J. Appl. Mech., Vol. 107, June 1985, pp. 446-452.
12. Bajaj, A. K., "Resonant Parametric Perturbations of the Hopf Bifurcation," J. Math. Anal. & Appl., Vol. 115, April 1986, pp. 214-224.
13. Bajaj, A. K., "Bifurcations in a Parametrically Excited Nonlinear Oscillator," Int. J. Nonlinear Mech., Vol. 22, 1987, pp. 47-59.
14. Streit, D. A., Krousgrill, C. M. and Bajaj, A. K., "A Preliminary Study of the Dynamic Stability of Flexible Manipulators Performing Repetitive Tasks", ASME J. Dynamic Syst., Meas. and Cont., Vol. 108, Sept. 1986, pp. 206-214.
15. Bajaj, A. K., "Nonlinear Dynamics of Tubes Carrying a Pulsatile Flow", Dynamics and Stability of Systems, Vol. 2, 1987, pp. 19-41.

16. Farhang, K., Midha, A. and Bajaj, A. K., "A Higher-Order Analysis of Basic Linkages for Harmonic Motion Generation", ASME J. Mechanisms, Transmissions, and Automation in Des., Vol. 109, Sept. 1987, pp. 301-307.

17. Farhang, K., Midha, A. and Bajaj, A. K., "Synthesis of Harmonic Motion Generating Linkages, Part I: Function Generation", ASME J. Mechanisms, Transmissions, and Automation in Des., Vol. 110, March 1988, pp. 16-21.

18. Heiman, M. S., Sherman, P. J. and Bajaj, A. K., "On the Dynamics and Stability of an Inclined Impact Pair", J. Sound and Vibration, Vol. 114, No. 3, 1987, pp. 535-547.

19. Streit, D. A., Krousgrill, C. M. and Bajaj, A. K., "Nonlinear Response of Flexible Robotic Manipulators Performing Repetitive Tasks", ASME J. Dynamic Syst., Meas. and Contr., Vol. 111, No. 3, 1989, pp. 470-480.

20. Huang, Y. M., Krousgrill, C. M. and Bajaj, A. K., "Nonlinear Response of a Dynamic System due to Oscillatory Flow", ASME J. Offshore Mech. and Arct. Engr., Vol. 109, No. 4, 1987, pp. 345-356.

21. Streit, D. A., Bajaj, A. K., and Krousgrill, C. M., "Combination Parametric Resonance Leading to Periodic and Chaotic Response in Two Degree-of-Freedom Systems with Quadratic Nonlinearities", J. Sound and Vibration, Vol. 124, No. 2, July 1988, pp. 297-314.

22. Tousi, S., Bajaj, A. K. and Soedel, W., "On the Stability of a Flexible Vehicle Controlled by a Human Pilot", Vehicle System Dynamics, Vol. 17, 1988, pp. 37-56.

23. Heiman, M. S., Bajaj, A. K. and Sherman, P. J., "Periodic Motions and Bifurcations in Dynamics of an Inclined Impact Pair", J. Sound and Vibration, Vol. 124, No. 1, July 1988, pp. 55-78.

24. Johnson, J. M. and Bajaj, A. K., "Amplitude Modulated and Chaotic Dynamics in Resonant Motion of Strings", J. Sound and Vibration, Vol. 128, January 1989, pp. 87-107.

25. Huang, Y. M., Krousgrill, C. M. and Bajaj, A. K., "Dynamic Behavior of Offshore Structures with Bilinear Stiffness", J. Fluids and Structures, Vol. 3, July 1989, pp. 405-422.

26. Bajaj, A. K. and Johnson, J. M., "Asymptotic Techniques and Complex Dynamics in Weakly Nonlinear Forced Mechanical Systems", Int. J. Non-Linear Mech, Vol. 25, No. 2/3, 1990, pp. 211-226.

27. Huang, Y. M., Bajaj, A. K. and Krousgrill, C. M., "Complex In-Line and Whirling Response of Structures to Oscillatory Flow", J. Sound and Vibration, Vol. 136, No. 3, 1990, pp. 491-505.

28. Tousi, S., Bajaj, A. K. and Soedel, W., "Finite Disturbance Closed-Loop Directional Stability of Vehicles with Human Pilot Considering Nonlinear Cornering Behavior", Vehicle System Dynamics (to appear).

29. Bajaj, A. K. and Tousi, S., "Torus Doublings and Chaotic Amplitude Modulations in a Two Degree-of-Freedom Resonantly Forced Mechanical System", Int. J. Non-Lin. Mech., Vo. 25, No. 6, 1990, pp. 625-642.

30. Tousi, S., Bajaj, A. K. and Soedel, W., "Closed-Loop Directional Stability of Car-Trailer Combinations", Vehicle System Dynamics (submitted).

31. Mahyuddin, A. I., Midha, A. and Bajaj, A. K., "On Methods for Evaluation of Parametric Stability and Response of Flexible Cam-Follower Systems", ASME J. Mech. Design (submitted).

32. Restuccio, J., Krousgrill, C. M. and Bajaj, A. K., "Nonlinear Nonplanar Dynamics of a Parametrically Excited Inextensional Elastic Beam", Nonlinear Dynamics (to appear).

33. Bajaj, A. K. and Sethna, P. R., "Effect of Symmetry-Breaking Perturbations on Flow-Induced Oscillations in Tubes", J. of Fluids and Structures (submitted).

34. Happawana, G., Bajaj, A. K. and Nwokah, O.D.I., "A Singular Perturbation Perspective on Mode Localization", J. Sound and Vibration (to appear).

35. Bajaj, A. K. and Johnson, J. M., "On the Amplitude Dynamics and "Crisis" in Resonant Motion of Stretched Strings", Proceedings of the Royal Society of London (submitted).

36. Happawana, G., Bajaj, A. K. and Nwokah, O.D.I., "A Singular Perturbation Analysis of Eigenvalue Veering and Mode Localization in Perturbed Linear Chain and Cyclic Systems", J. Sound and Vibration (submitted).

Conference Publications:

1. Sethna, P. R. and Bajaj, A. K., "Bifurcation in Nonlinear Oscillatory Systems," Proc. of VIIIth Int. Conference on Nonlinear Oscillations, Prague, 1978, pp. 621-626.

2. A. K. Bajaj, "Primary Parametric Resonance in a Self-Excited Oscillator," Proceedings of the XVIIIth Midwestern Mechanics Conference, The University of Iowa, 1983.

3. A. K. Bajaj, "Some Interactions in Flow-Induced Motions of Tubes," Proceedings of the ASCE, Engineering Mechanics Specialty Conference, Purdue University, 1983.

4. Streit, D. A., Krousgrill, C. M. and Bajaj, A. K., "Dynamic Stability of Flexible Manipulators Performing Repetitive Tasks", in Robotics and Manufacturing Automation, PED-Vol. 15, eds. M. Donath, M. Leu, ASME, New York, 1985.

5. Heiman, M. S., Sherman, P. J. and Bajaj, A. K., "On the Dynamics and Stability of an Inclined Impact Pair", Proceeding of the 27th AIAA/ASME/ASCE/AHS Structures, Structural Dynamics and Materials Conference, San Antonio, TX, May 19-21, 1986, pp. 585-591. Paper No. AIAA-86-0998-CP.

6. Farhang, K., Midha, A. and Bajaj, A. K., "A Higher-Order Analysis of Basic Linkages for Harmonic Motion Generation", ASME Paper 86-DET-70, 19th Biennial ASME Mechanisms Conference, Columbus, Ohio, October 1986.

7. Bajaj, A. K. and Krousgrill, C. M., "Complex Nonlinear Dynamics of a Rotating Pendulum", Proceedings of the 20th Midwestern Mechanics Conference, Purdue University, Aug. 31 - Sept. 2, 1987, pp. 832-839.

8. Johnson, J. M. and Bajaj, A. K., "Chaotic Amplitude Modulations in Forced Vibrations of Strings", Proceedings of the 20th Midwestern Mechanics Conference, Purdue University, Aug. 31 - Sept. 2, 1987, pp. 851-856.

9. Tousi, S., Bajaj, A. K. and Soedel, W., "On Stability of Flexible Vehicle Controlled by a Human Pilot", Proceedings of the 20th Midwestern Mechanics Conference, Purdue University, Aug. 31 - Sept 2, 1987, pp. 1354-1359.

10. Huang, Y. M., Krousgrill, C. M., and Bajaj, A. K., "Subharmonic Resonances in a Bilinear Dynamic System due to Oscillatory Flow", Proceedings of the 20th Midwestern Mechanics Conference, Purdue University, Aug. 31 - Sept. 2, 1987, pp. 637-642.

11. Bajaj, A. K., "Bifurcations in Flow-Induced Oscillation in Tubes Carrying a Fluid", Dynamical Systems Approaches to Nonlinear Problems in Systems and Circuits, eds. Fathi M. A. Salam and Mark L. Levi, SIAM, Philadelphia, 1988.

12. Farhang, K., Midha, A. and Bajaj, A. K., "Synthesis of Harmonic Motion Generating Linkages, Part I: Function Generation", Advances in Design Automation - 1987, DE-Vol. 10-2, ed. S.S. Rao, ASME, New York, 1987.

13. Streit, D. A., Bajaj, A. K. and Krousgrill, C. M., "Parametric Instabilities and Chaotic Amplitude Modulations in a System with Two Degrees-of-Freedom", Proceedings of 3rd International Conf. on Recent Advances in Structural Dynamics, University of Southampton, Southampton, July 1988, pp. 809-818.

14. Huang, Y. M., Krousgrill, C. M. and Bajaj, A. K., "Dynamic Behavior of Offshore Structures with Bilinear Stiffness", 1988 International Symposium on Flow-Induced Vibration and Noise, Vol. 7 - Nonlinear Interaction Effects and Chaotic Motions, eds: M. M. Reischman, M. P. Paidoussis, R. J. Hansen, ASME, NY 1988.

15. Zadoks, R. I. and Bajaj, A. K., "Mel'nikov's Criterion and Chaos in a Model of an Arch", Proceedings of the First Pan-American Congress of Applied Mechanics, Rio de Janeiro, Brazil, January 3-6, 1989.

16. Mahyuddin, A. I., Midha, A. and Bajaj, A. K., "On Methods for Evaluation of Parametric Stability and Response of Flexible Cam-Follower Systems", ASME 1990 Mechanisms Conference, Chicago, IL, Sept. 15-18, 1990.

17. Chang, S. I., Bajaj, A. K. and Krousgrill, C. M., "Amplitude Modulated Dynamics in Harmonically Excited Non-Linear Oscillations of Rectangular Plates with Internal Resonance", Accepted for 4th Int. Conference on Recent Advances in Structural Dynamics, Inst. of Sound and Vibration Research, Southampton, England, July 1991.

18. Happawana, G., Bajaj, A. K. and Nwokah, O.D.I., "Mode Localization in Structural Systems: A Singular Perturbation Analysis", Submitted for 32nd AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference, Baltimore, Maryland, April 8-10, 1991.

19. Bajaj, A. K., "Examples of Crisis Phenomenon in Structural Dynamics", Accepted for Proceedings of the 1990 Conference on Bifurcations and Chaos; Theory, Algorithms and Applications, Würzburg, W. Germany.

20. Happawana, G., Bajaj, A. K. and Nwokah, O.D.I., "A Singular Perturbation Analysis of Eigenvalue Veering and Mode Localizaiton in Perturbed Linear Chain and Cyclic Systems", Submitted for the 13th Biennial ASME Conference on Mechanical Vibration and Noise, Miami, Florida, September 22-25, 1991.

21. Happawana, G., Nwokah, O.D.I. and Bajaj, A. K., "On the Dynamics of Perturbed Symmetric Systems", Submitted for the 13th Biennial ASME Conference on Mechanical Vibration and Noise, Miami, Florida, September 22-25, 1991.

22. Mohamad, A.A. and Bajaj, A. K., "Dynamics of a Spinning Toroidal Thermosyphon", Accepted for the 1991 ASME/AIChE National Heat Transfer Conference, Minneapolis, Minnesota, July 28-31, 1991.

Conference Presentations:

1. "Bifurcations in Dynamical Systems with Internal Resonance," P. R. Sethna, A. K. Bajaj, ASME Winter Annual Meeting, San Francisco, CA, December 10-15, 1978.

2. "Bifurcations in Three-Dimensional Motions of Articulated Tubes," A. K. Bajaj, P. R. Sethna, IXth U.S. National Congress of Applied Mechanics, Cornell University, Ithaca, NY, June 21-25, 1982.

3. "Bifurcations in a Parametrically Excited Nonlinear Oscillator", A. K. Bajaj, 16th ICTAM held at Lyngby, Denmark, August 19-25, 1984.

4. "Period Doubling Bifurcations and Modulated Motions in Forced Mechanical Systems", S. Tousi, A. K. Bajaj, 21st Annual Meeting of the Society of Engineering Science, VPISU, Blacksburg, Virginia, October 15-17, 1984.

5. "Nonlinear Dynamics of Tubes Carrying a Pulsatile Flow", A. K. Bajaj, 21st Annual Meeting of the Society of Engineering Science, VPISU, Blacksburg, Virginia, October 15-17, 1984.

6. "Parametric Excitation, Period Doubling and Chaos in Two-Degree-of-Freedom Flexible Mechanical Systems", D. A. Streit, A. K. Bajaj, C. M. Krousgrill, 10th U.S. National Congress of Applied Mechanics, The University of Texas at Austin, TX, June 16-20, 1986.

7. "Nonlinear Dynamics, Resonances and Chaotic Motions in Harmonically Excited Mechanical Systems", A. K. Bajaj, 23rd Annual Meeting of the Society of Engineering Science, SUNY, Buffalo, N.Y., August 25-27, 1986.

8. "Bifurcations in Flow Induced Oscillations in Tubes Carrying a Fluid", A. K. Bajaj, Engineering Foundation Conference on "Qualitative Methods for the Analysis of Nonlinear Dynamics", Henniker, N.H., June 8-13, 1986.

9. "Stability and Nonlinear Dynamics of Flexible Manipulators Performing Repetitive Tasks", A. K. Bajaj, C. M. Krousgrill, D. A. Streit, AFOSR/ARO Conference on Non-Linear Vibrations Stability, and Dynamics of Structures and Mechanisms, VPISU, Blacksburg, March 23-25, 1987.

10. "Nonlinear Dynamic Response of a Structure Due to Oscillatory Flow", Y. M. Huang, C. M. Krousgrill and A. K. Bajaj, 11th Canadian Congress of Applied Mechanics, Edmonton, May 31 - June 4, 1987.

11. "Periodic Motions and Bifurcations in Dynamics of an Inclined Impact Pair", M. S. Heiman, P. J. Sherman and A. K. Bajaj, 11th Canadian Congress of Applied Mechanics, Edmonton, May 31 - June 4, 1987.

12. "Flow-Induced, Bi-Planar Motion of Marine Structures", Y. M. Huang, A. K. Bajaj, C. M. Krousgrill, Applied Mechanics and Engineering Science Conference, Berkeley. June 20-22, 1988.

13. "Amplitude Modulated Chaos in Harmonically Excited Mechanical Systems", A. K. Bajaj, J. M. Johnson, SIAM 1988 Annual Meeting, Minneapolis, Minnesota, July 11-15, 1988.

14. "Chaotic Amplitude Dynamics of an Autoparametric Two-Degree-of-Freedom System", A. K. Bajaj, J. M. Johnson, Accepted for IUTAM Symposium on Nonlinear Dynamics in Engineering systems, University of Stuttgart, Stuttgart, FRG, August 21-25, 1989.

15. "Nonlinear Dynamics of a Parametrically Excited Inextensional Elastic Beam", J. M. Restuccio, C. M. Krousgrill and A. K. Bajaj, SIAM Conference on Dynamical Systems, Orlando, FL., May 7-10, 1990.

16. "On the Dynamics of Perturbed Symmetric Systems", G. Happawana, O.D.I. Nwokah, D. Afolabi and A. K. Bajaj, 11th U.S. National Congress of Applied Mechanics, Univ. of Arizona, Tucson, May 21-25, 1990.

Invited Conference Presentations

1. "Asymptotic Techniques and Chaos in Weakly Nonlinear Forced Mechanical Systems", A. K. Bajaj, Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanisms Conference, VPISU, Blacksburg, June 1-3, 1988.

2. "Torus Doubling and Chaotic Amplitude Modulations in a Forced Mechanical System", A. K. Bajaj and S. Tousi, Inter. Conf. on Dynamical Systems, Control Theory and Applications, Wright State University, Dayton, June 15-17, 1989.

3. "Examples of Crisis Phenomena in Structural Dynamics", A. K. Bajaj, Joint ASCE/ASME Mechanics Conference, San Diego, July 10-12, 1989.

4. "On The Complex Whirling Dynamics of Strings", A. K. Bajaj, Third Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms, VPISU, Blacksburg, June 25-27, 1990.

5. "On the Complex Whirling Dynamics of Strings", A. K. Bajaj, Conference on Bifurcation and Chaos: Analysis, Algorithms and Applications, Würzburg, West Germany, August 20-24, 1990.

Invited Talks:

1. "Bifurcations in Systems with Symmetry," given in Control Science and Dynamical System Center, University of Minnesota, Minneapolis, MN, January 29, 1981.

2. "Resonant Parametric Perturbations of the Hopf Bifurcation," given at the University of Missouri-Rolla, Rolla, MO 65401, on February 25, 1983, jointly sponsored by the Departments of Engineering Mechanics and Mechanical Engineering.

3. "Some Examples of Complex Dynamics in Mechanical Systems", given in Department of Mechanical Engineering, I.I.T. Kanpur, India, on August 5, 1985.

4. "Internal Resonance and Chaos in Externally Excited Mechanical Systems" given in Department of Aeronautical and Astronautical Engineering, University of Illinois at Urbana-Champaign, November 24, 1986.

5. "Chaotic Amplitude Modulations in Nonlinear Systems - Examples from Mechanical Oscillators", given in School of Chemical Engineering, Purdue University, West Lafayette, October 1, 1987.

6. "Chaotic Amplitude Modulations in Harmonically Excitated Weakly Nonlinear Systems", given at Indian Institute of Technology, Kanpur, India, March 30, 1988.

7. "Introduction to Bifurcations and Chaos", given at Indian Institute of Technology, Kharagpur, India, April 4, 1988.

8. "Chaotic Dynamics of Impacting Systems", given at Indian Institute of Technology, Kanpur, India, April 7, 1988.

9. On the Complex Resonant Motions of Structures and Whirling Dynamics of Strings" given at Department of Mechanical Engineering, University of Pittsburgh, Pittsburgh, October 23, 1990.

Book Reviews:

Non-Linear Oscillations, by P. Hagedorn, Oxford Univ. Press, Mechanism and Machine Theory Vol. 20, No. 3, 1985, pp. 243.

Others:

1. Attended Conference on "New Approaches to Nonlinear Problems in Dynamics," sponsored by Engineering Foundation at Asilomar Conference Grounds, Pacific Grove, CA, December 9-14, 1979.
2. Attended Workshop on "NSF Research Needs in Theoretical Foundation of Dynamics", State University of New York at Buffalo, August 24, 1986.
3. Session Chairman, "Chaotic Motions" in 20th Midwestern Mechanics Conference, Purdue University, Aug. 31-Sept. 1, 1987.
4. Member, Organizing Committee, 20th Midwestern Mechanics Conference, Purdue University, Aug. 31 - Sept. 1, 1987.
5. Participated in Presenting four-day short course "Applied Digital Signal Processing", Purdue University, October 1987.
6. Attended Short Course "Nonlinear Dynamics, Chaos, and Bifurcation", SIAM Annual Meeting, Minneapolis, July 10, 1988.
7. Participated in Presenting five-day short course" Applied Digital Signal Processing", Purdue University, June 26-30, 1989.
8. Participated in Presenting five-day short course "Applied Digital Signal Processing", Purdue University, June 18-22, 1990.
9. Session Chairman, "Methods I" at the "3rd Conference on Nonlinear Vibrations, Stability, and Dynamics of Structures and Mechanisms", VPISU, Blacksburg, June 25-27, 1990.

Other Publications:

Midha, A. annd Bajaj, A. K., "The Convolution Integral Solution of Linear Spring-Mass-Damper Vibration System -- A Versatile Tool", Mech. Eng'g. News, Vol, 23, No. 3, 1986, pp. 5-15.

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General Record

1. Education

PhD 1983 (Mechanical Engineering) Imperial College, London, England.

MSc 1978 (Mechanical Engineering) Imperial College, London, England.

BSc 1976 (Mechanical Engineering) Thames Polytechnic, London, England.

2. Academic Appointments

Visiting Scientist, Summer 1990
Massachusetts Institute of Technology, Cambridge, Massachusetts.

Visiting Scientist, Summer 1989
Institute for Computational Mechanics in Propulsion,
NASA Lewis Research Center, Cleveland, Ohio.

Associate Professor of Mechanical Engineering, July1989-present
Indiana University-Purdue University at Indianapolis.

Assistant Professor of Mechanical Engineering, September 1985-June 1989
Indiana University-Purdue University at Indianapolis.

Research Associate, April 1984-August 1985
University of Maryland, College Park, Maryland.

Research Student, September 1979-December 1982
Imperial College, London, England.

3. Awards and Honors

Rolls-Royce Scholar at Imperial College, 1979-1982.

Awarded a *Mark of Distinction* by the University of London Senate, 1978.

Awarded BP (British Petroleum Co.) Scholarship at Thames Polytechnic, London, 1972-1976.

4. Membership in Professional Societies

Member, American Institute of Aeronautics and Astronautics (AIAA).

Member, American Society of Mechanical Engineers (ASME).

Member, Society for Experimental Mechanics (SEM).

Member, International Gas Turbines Institute.

Research Record

1. Research Interests

Vibration of Structures (including large scale structures) • Classical Dynamics • Dynamical Systems (including Bifurcation of Vector Fields and Chaos) • Singularity Theory and Catastrophe Theory • Computer Aided Testing (including Experimental Modal Analysis) • Parallel Computation (mainframe — Cray XMP, YMP —and desktops computers — Transputers, Hypercube systems, etc) • Applied Mathematics and Numerical Methods in Engineering (including Finite Element Method).

2. Publications

a. Articles in Journals, Monographs and Books

Afolabi, D., and Crandall, S. H., 1991, "The Weyl Groups A_k, D_k, E_k and the Random Vibration of Symmetric Structures" (forthcoming).

Nwokah, O. D. I., Afolabi, D., and Damra, F. M., 1990, "On the Modal Stability of Imperfect Cyclic Systems", in *Control and Dynamic Systems: Advances in Theory and Application*, vols 35, Part 2, pp 137-164. Edited by C. T. Leondes, Academic Press.

Afolabi, D., 1990, *On the Geometric Stability of Certain Modes of Vibration*, NASA Technical Memorandum (forthcoming).

Afolabi, D., 1989, *Effects of Mistuning and Matrix Structure on the Topology of Frequency Response Curves*, NASA Technical Memorandum: TM-102290. National Aeronautics and Space Administration.

Afolabi, D., 1988, "A Note on the Rogue Failure of Turbine Blades", *Journal of Sound and Vibration*, vol 122, pp 535-545.

Afolabi, D., 1988, "Vibration Amplitudes of Mistuned Blades", ASME *Journal of Turbomachinery*, vol 110, pp 251-257.

Afolabi, D., 1987, "Linearization of the Quadratic Eigenvalue Problem", *Computers and Structures*, vol 25, pp 1039-1040.

Afolabi, D., 1987, "The Dynamic Stiffness Method in Vibration Analysis", *International Journal of Mechanical Engineering Education*, vol 15, pp 133-139.

Afolabi, D., 1986, "Natural Frequencies of Cantilever Blades with Resilient Roots", *Journal of Sound and Vibration*, vol 110, pp 429-441.

Afolabi, D., 1985, "The Frequency Response of Mistuned Bladed Disk Assemblies", in *Vibrations of Blades and Bladed Disk Assemblies*, edited by R. E. Kielb and N. F. Rieger, American Society of Mechanical Engineers, New York.

Afolabi, D., 1985, "The Eigenvalue Spectrum of a Mistuned Bladed Disk", in *Vibrations of Blades and Bladed Disk Assemblies*, edited by R. E. Kielb and N. F. Rieger, American Society of Mechanical Engineers, New York.

b. Articles in Conference Proceedings

Afolabi, D., 1991, "Modes and Quasi-Modes in Modal Analysis", Proceedings, 9th International Modal Analysis Conference, April 15-18, 1991, Florence, Italy.

Afolabi, D., and Alabi, B., 1991, "Resolution of Double Modes", Proceedings, 9th International Modal Analysis Conference, April 15-18, 1991, Florence, Italy.

Afolabi, D., 1991, "Versal Deformation of Degenerate Frequency Response Functions", Proceedings, 4th Workshop on Control and Dynamic Systems, University of Southern California, Los Angeles, January, 1991.

Afolabi, 1991, "Small Denominators and the Problem of Eigenvector Stability in Structural Dynamics" Midwestern Mechanics Conference, 1991.

Alabi, B., and Afolabi, D., 1990, "On the Problem of Uniformly Moving Tangential Loads on a Semi-infinite Solid", Proceedings of the 11th U.S. National Congress of Applied Mechanics, Tucson, AZ.

Happawana, G. S., Nwokah, O. D. I., Afolabi, D., and Bajaj, A. K., 1990, "On the Dynamics of Perturbed Symmetric Systems", Proceedings of the 11th U.S. National Congress of Applied Mechanics, Tucson, AZ.

Afolabi, D., and Nwokah, O.D.I. 1990, "Effect of Mild Perturbations on the Dynamics of Structures with Circulant Matrices", Presented at the 1990 AIAA/ASME/ASCE/AHS Structural Dynamics & Materials Conference, Longbeach, CA, April 2-4.

Afolabi, D., and Nwokah, O. D. I., 1989, "The Frequency Response of Mistuned Cyclic Systems", Proceedings of the 12th ASME Biennial Vibrations Conference, Montreal, Canada, September 17-21.

Afolabi, D., and Nwokah, O. D. I., 1989, "On the Modal Stability of Imperfect Cyclic Systems", Proceedings of the 2nd Workshop on Control Mechanics, University of Southern California, Los Angeles, January 25-27.

Afolabi, D., and Nwokah, O. D. I., 1989, "Dynamics and Control of Perturbed Periodic Systems", Proceedings of the 21st Midwestern Mechanics Conference, Michigan Technological University, August 13-16, Houghton, MI. (Also published in *Developments in Mechanics*, vol 15, pp 539-540.

Paydar, N. and Afolabi, D., 1989, "Buckling Analysis of Tapered Sandwich Plates", Proceedings of the 21st Midwestern Mechanics Conference, Michigan Technological University, Houghton, MI. (Also published in *Developments in Mechanics*, vol 15, pp 195-196.

Afolabi, D., 1988, "Modal Characteristics of Turbomachine Blades in a Multi-Stage Engine", Proceedings of the 6th International Modal Analysis Conference, Orlando, Florida, February 1-4, pp 521-527.

Afolabi, D., 1987, "An Anti-Resonance Technique for Detecting Structural Damage", Proceedings of the 5th International Modal Analysis Conference, Imperial College, London, England, April 6-9, pp 491-495.

Afolabi, D. and Paydar, N., 1987, "Receptance Analysis of Timoshenko Beams with Various End Conditions", Proceedings of the 20th Midwestern Mechanics Conference, Purdue University, West Lafayette, Indiana, August 31-September 2. (Also published in *Developments in Mechanics*, vol 14c, pp 1230- 1235.

c. Internal Reports

Afolabi, D., 1988, "Forced Response Analysis of Bladed Disk Assemblies", Final Report to: Allison Gas Trubines, General Motors Corp., Indianapolis (20 pages).

Afolabi, D., 1988, "BDA — User Guide to the Computer Program for Blade Mistuning Analysis", Contract Report to Gas Trubines, General Motors Corp., Indianapolis (31 pages).

Afolabi, D., 1982, "Some Vibration Characteristics of an Aeroengine Compressor Fan", Contract Report No 8202, Mechanical Engineering Department, Dynamics Lab, Imperial College, London, England Report to: Rolls-Royce Research Center, Derby, England (23 pages).

Afolabi, D., 1982, "Effects of Random Mistuning on the Vibration of Coupled Turbomachine Blades", Dynamics Lab Report No 8201, Mechanical Engineering Department, Imperial College, London, England (85 pages).

Afolabi, D., 1982, "A Statistical Interpretation of the Dynamic Response Data of Bladed Disc Assemblies", Dynamics Lab Report No 82016, Mechanical Engineering Department, Imperial College, London, England (42 pages).

Afolabi, D. and Ewins, D. J., 1979, "A Bibliography of Blade and Disk Vibration", Dynamics Lab Report No 7919, Mechanical Engineering Department, Imperial College, London, England (132 pages).

Afolabi, D. and Ewins, D. J., 1978, "The Vibration of Turbine Blade Packets", Dynamics Lab Report No 78007, Mechanical Engineering Department, Imperial College, London, England (156 pages).

d. Publications in Review

Afolabi, D., *Modal Interaction in Dynamic Systems*, NASA Technical Memorandum (forthcoming, 1991).

e. Works in Progress

See attachment.

3. Invited Lectures

1991 The Mathematical Institute of Oberwolfach, West Germany, "The Weyl Groups A_k, D_k, E_k and the Random Vibration of Symmetric Structures" ; forthcoming, 1991. (Co-author: S. H. Crandall.)

1989 NASA Lewis Research Center, Institute for Computational Mechanics in Propulsion, July 1989: "Qualitative Characteristics of Mistuned Cyclic Systems".

1989 Hughes Aircraft, Division of General Motors Corporation, El Segundo, CA, January 28, 1989: "Dynamics and Control of Perturbed Periodic Systems".

1988 NASA Lewis Research Center, Workshop on *Unsteady Phenomena in Turbomachinery*, Wednesday, July 20: "Effect of Aerodynamic and Structural Inter-Stage Coupling on the Vibration of Mistuned Blades".

1988 NASA Lewis Research Center, Structural Dynamics Branch Seminar Series June 9: "Effect of Mistuning on Mode Localization and Mode Splitting in a Multi-Stage Engine".

1988 United Technologies Research Center, Pratt & Whitney Aircraft, East Hartford, Connecticut, March 24: "A Systems Approach for Blade Mistuning".

1988 Purdue University, School of Mechanical Engineering, West Lafayette Indiana, September 29: "Catastrophe Theory and the Dynamics of Imperfect Cyclic Systems".

1987 Allison Gas Turbines Division, General Motors Corporation, November: "Some Recent Advances in Turbine Blade Vibration".

4. Consulting Activities

1988-present: NASA Lewis Research Center, Cleveland, Ohio.

1987-89: Allison Gas Turbines Division, General Motors Corporation, Indianapolis, Indiana.

1988: United Tecnnologies Research Center, East Hartford, Connecticut.

4. Research Grants

a. Internal Grants—*Principal Investigator/Grantee*: Daré Afolabi

1989: Indiana University, Bloomington, Indiana, *Project Development Grant.*

1988: Purdue University, West Lafayette, Indiana, *XL Summer Grant.*

1987: Indiana University, Bloomington, Indiana, *Overseas Conference Grant.*

1987: Indiana University, Bloomington, Indiana, *Project Development Grant.*

1986: Indiana University, Bloomington, Indiana, *Summer Faculty Fellowship.*

b. External Grants Awarded—*Principal Investigator*: Daré Afolabi

1988-90: from the Air Force Office of Scientific Research, AFOSR, Bolling Air Force Base, Washington DC. *Dynamics and Control of Bladed Disk Assemblies* [November, 1988 - October, 1989].

1987: from General Motors Corporation, Allison Gas Turbines Division, Indianapolis, Indiana. *Forced Response of Mistuned Turbine Blades* [January, 1988 - December, 1988].

6. Review Activities

Reviewer for the National Science Foundation.

Reviewer for McGraw-Hill Book Company.

Reviewer for *Journal of Sound and Vibration.*

Reviewer for ASME *Journal of Vibration and Acoustics.*

Reviewer for ASME *Journal of Engineering for Gas Turbines and Power.*

Reviewer for ASME *Journal of Turbomachinery.*

Service Record

1. University Service

a. Campus-wide

Member, Faculty Development Council [1989-90].

Vice-President, Black Faculty and Staff Council [1988-89].

Volunteer, "Person-to-Person Week", April, 1988.

Member, Campus Interrelations Sub Committee [1987-88].

b. School of Engineering and Technology

Member, Committee for the Development of Academic Computing Plan, [1987-88].

Member, Faculty-Student Advisory Committee [1989-90].

c. Department of Mechanical Engineering

Member, Search and Screen Committee [1987-89].

Coordinator, Seminars and Colloquia Series [1986-1987].

Coordinator, Laboratory Equipment and Instrumentation [1988].

d. Student Support Service

Academic Counselor, Mechanical Engineering Department.

Advisor, IUPUI Chapter, National Society of Black Engineers.

Mentor, Minority Engineering Advancement Program.

2. Public Service

Judge, City of Indianapolis Section, National MATHCOUNTS Competition for Pre-College students, February 20, 1988.

Judge, ACTSO-87 and ACTSO-88 Science and Engineering Competitions for High School students, April 1987 and June 1988